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CONTENTS

ECONOMIC FACTORS IN INFANT MORTALITY. By ROBERT M. WOODBURY 137

SEASONAL VARIATION AS A RELATIVE OF SECULAR TREND. By LINCOLN W. HALL 156

THE MEASUREMENT OF SEASONAL VARIATION. By HELEN D. FALKNER 167

THE PROBLEM OF A STANDARD INDEX NUMBER FORMULA. By GEORGE R. DAVIES 180

A NEW INDEX OF GENERAL PRICE LEVEL. By CARL SNYDER 189

THE NATURE OF AN UNEQUIVOCAL PRICE-INDEX AND QUANTITY-INDEX. PART II. By SIR GEORGE H. KNIBBS 196

NOTES:

TEN YEARS' EXPERIENCE IN BUSINESS STATISTICS. By H. Parker Willis 206

A GENERALIZED LAW OF ERROR. By Paul R. Rider 217

DEATH RATES FROM PUERPERAL SEPTICEMIA AND THEIR VARIABILITY. By G. E. Harmon 221

COTTON STATISTICS FROM THE DEPARTMENT OF COMMERCE (226); PROGRESS OF WORK IN THE CENSUS BUREAU (231); THE COLLECTION OF ORIGINAL STATISTICAL DATA (232); THE LEAGUE OF NATIONS HEALTH SERVICE (234); MISCELLANEOUS NOTES (236).

REVIEWS: Moulton and McGuire: *Germany's Capacity to Pay: A Study of the Reparations Problem*, by A. A. Young (242); Newsholme: *The Elements of Vital Statistics in Their Bearing on Social and Public Health Problems*, by W. F. Willcox (246); East: *Mankind at the Crossroad*, by Louis I. Dublin (249); Fisher: *Wage Rates in the Coal Industry*, and Bezanson: *Earnings of Bituminous Coal Miners*, by H. K. Herwitz (253); Clark: *Studies in the Economics of Overhead*, by W. A. Paton (257); Vanderblue and Burgess: *Railroads: Rates—Service—Management*, by J. M. Clark (259); Hastings: *Costs and Profits: their Relation to Business Cycles*, by J. Viner (262); Pearl: *Introduction to Medical Biometry*, by C. B. Davenport (264); Schluter: *The Pre-War Business Cycle*, by W. A. Berridge (265); Huntington and Visher: *Climactic Changes. Their Nature and Causes*, by E. W. Kopf (266).

RECENT LITERATURE 269

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ECONOMIC FACTORS IN INFANT MORTALITY[1]

By Robert Morse Woodbury

Many writers upon infant mortality have pointed out the correlation that exists between high infant death rates and low economic position.[2] Such a correlation, however, does not necessarily imply a causal connection; for example, if a much larger proportion of the babies in the low than in the high income groups were artificially fed, such a correlation might be merely a reflection of a high mortality among artificially fed infants. Or, if a larger proportion of the babies in the low than in the high income groups belonged to races that were characterized by high infant death rates, such a correlation might be merely a reflection of the high rates prevailing among these races. Or, again, if a larger proportion of births in the lowest income group were fifth or later in order and high orders were associated with a high infant death rate, the correlation with order of birth might account for part of the relationship between low income and high infant mortality.

On the other hand, the correlation between income and infant death rates may be found to be explained in part by correlations between such rates and employment of the mother, housing congestion and other economic factors. In these cases, if it is shown that employment of the mother and housing congestion are themselves causally influenced by low income, the responsibility of low economic position for high infant mortality is not lessened by the presence of intermediate factors in the chain of causation.

[1] A summary of a chapter on "Economic Factors in Infant Mortality" to be published in *Causal Factors in Infant Mortality* by the Children's Bureau, U. S. Department of Labor.

[2] See, for example, Prinzing, F., *Handbuch der medizinischen Statistik*, pp. 300–304.
Westergaard, H. "Die Lehre von der Mortalität und Morbilität, *Anthropologisch-Statistische Untersuchungen*, Jena, 1902, pp. 394–402.
Hibbs, H. H., Jr. *Infant Mortality: Its relation to social and industrial conditions*, pp. 121–127, and others cited infra.

Evidently, therefore, a thorough study of the relation between poverty and infant mortality requires material for and methods of analysis that permit of distinguishing between these and other different possible explanations of any correlation found between the two.

The information available from birth and death certificates which bears upon the economic position of families in which children are born is limited to the father's occupation, from which inferences may be drawn as to his economic status. On this point an analysis of the English statistics for 1911 shows that the mortality rate for legitimate infants in the upper and middle class (76.4) was much lower than in the intermediate (106.4) or in the wage-earning (132.5) class.[1] But not only are the occupations difficult to classify according to economic status in many instances, but the procedure of treating all persons having the same occupation as if they had the same economic position tends to obscure any effect which economic position may have. Furthermore, with the limited information available, the elimination of the influence of other factors, such as prevalence of artificial feeding or of a high birth rate, is difficult if not impossible.

In most studies of the subject, therefore, recourse has been had to some indirect method of measuring the influence of economic position. Comparative figures of infant mortality for the different wards of a city or for different cities or districts are correlated with indices of economic status for the same areas. As indices have been used, for example, the average income,[2] the proportion of domestic servants in the female population,[3] and the proportion of persons in receipt of poor relief.[4] Not only are these indices relatively unsatisfactory but also, because of the indirectness of the method, the influence of economic position tends to be obscured and the elimination of other factors, such as type of feeding[5] or the "birth rate," becomes especially difficult.[6] [7]

[1] T. H. C. Stevenson, *74th Annual Report of the Registrar General of Births, Deaths and Marriages in England and Wales, 1911*, Cd. 6578, pp. xl–xlv, 73–92, especially p. 88.

[2] von Neefe, quoted in Prinzing, F., *Handbuch der medizinischen Statistik*, p. 302.

[3] Great Britain. *Thirty-ninth Annual Report of Local Government Board, 1909–1910.* Supplement to the Report of the Board's Medical Officer containing a report by the Medical Officer on Infant and Child Mortality. Cd. 5263, p. 55.

[4] Groth, Alfred, M. D., and Hahn, Prof. Martin, M. D., "Die Säuglingsverhältnisse in Bayern. *Zeitschrift d. k. Bayerischen-Statistischen Landesamts*, 42nd year, 1910, No. 1, pp. 78–164, especially p. 101.

[5] Differences in type of feeding as a possible explanation of the higher mortality found among the babies of the poor are eliminated in some of the studies by evidence that artificial feeding is less prevalent among the poor than among the rich. See, *Infant and Child Mortality* Cd 5263, p. 54–55: Prinzing, *Handbuch der medizinischen Statistik*, pp. 300–304.

[6] Greenwood, M., Jr., and Brown, J. W., in "An Examination of Some Factors Influencing the Rate of Infant Mortality," *Journal of Hygiene*, Vol. XII, pp. 5–45, applied to the material secured by Groth and Hahn for rural communes and urban districts in Bavaria the method of partial correlation to separate the influence of the economic factor from those of artificial feeding and the birth rate. Their

In the course of studies made in seven American cities by the Children's Bureau, records have been accumulated for over 21,000 legitimate live births that occurred between November 1, 1912, and December 31, 1915, for each of which data are available not only as to sex, single or plural birth, cause of death, and age at death—items which are commonly secured—but also as to type of feeding, race and nationality of mother, earnings of father, housing conditions, and employment of mother during pregnancy and during the first year of the infant's life. Separate reports dealing with the findings of each study have been issued and in each the relation found to exist between economic factors and infant mortality was pointed out; [1] but the combined data which are used as the basis for the present discussion permit of a much fuller analysis than was possible for the comparatively small numbers of births in the separate cities. This material offers, therefore, a comparatively large mass of data which permits not only of showing the direct correlation between the infant mortality rate and such economic factors as father's earnings, housing congestion, and mother's employment, but also of isolating the influence of each of these factors from that of the others and from that of such factors as type of feeding and race and nationality of mother.

It should be emphasized that the various measurements of social and economic conditions used in the analysis are in every case applied directly to the particular infant whose chances of life might be affected. Thus, the data on the father's earnings relate to the amount earned during the year immediately following the baby's birth, that is, during the period when these earnings would be associated most closely with the baby's chances of life; the information on the mother's employment relates to work during the period of pregnancy preceding and during the year following the infant's birth; and housing conditions relate to the house in which the family lived during the infant's first year.

The method of securing the data, so far as they could not be obtained from birth and death records, was by interview with the mother. On

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conclusion, based upon the rural communes alone, was a negative one — that the evidence did not show that the average infant mortality rate in the rural communes varied significantly (*i. e.* with respect to the probable error) with the average economic position of the commune as measured by the proportion of persons in receipt of relief, when prevalence of artificial feeding and the birth rate were held constant (p. 34).

[7] Reference should be made also to the correlation of cyclical changes in industrial conditions with changes in infant mortality rate, see W. F. Ogburn and Dorothy S. Thomas, "The Influence of the Business Cycle on Certain Social Conditions," JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION, Vol. XVIII, p. 335–337.

[1] For details as to findings and methods of study see the series of studies on *Infant Mortality in Manchester, N. H., Waterbury, Conn., Brockton, Mass., Saginaw, Mich., New Bedford, Mass., Akron, Ohio, and Baltimore, Md.*, U. S. Children's Bureau Publications Nos. 20, 29, 37, 52, 68, 72, 119.

most of the questions studied—the infant's feeding, nationality (mother tongue), housing conditions, etc., the mothers were in the best position to give satisfactory information. With regard to father's earnings the mothers were asked not only as to the total amount, but also as to wage rates, periods of unemployment and sickness, etc. If discrepancies were found—for example, if the time at work at the stated wages would not have yielded the total amount said to have been earned, or if the mother could not give the information—the data were secured later from the father. In case the father had died or had deserted previous to the birth of the infant or during the year following, the amount used was what he actually earned during that part of the year when he was contributing to the support of the family. In a few cases, therefore, in which the father deserted or died during the year, the amount of his earnings which the family had to live on was much below his normal earnings for a full year; and in other cases in which the father had died or deserted prior to the birth of the child, he was classified as having "no earnings." These cases were not numerous, but they are mentioned in order to emphasize the fact that the reports of earnings were not, and should not be interpreted as, statements either of yearly wage rates or of total family incomes.

In most of the cities studied the statements of earnings thus secured were not compared with pay-roll data nor was information obtained from employers. In the city of Manchester, however, in which a very large proportion of the fathers of infants included in the study were employed in the textile mills, the mothers' statements of fathers' earnings were compared with the amounts as shown on the pay-rolls. For this city it was found that, although on the whole the statements of the mothers tended to exaggerate slightly the amounts actually received,[1] when the data were classified according to the earnings as stated by the mothers, the results were substantially the same as when they were classified according to the amounts derived from the pay-rolls. The comparison of the results of the two methods of securing information as to earnings in this city seemed to indicate that the data secured in other cities by interviewing the mothers were substantially accurate.

Besides data on the father's earnings, information was secured as to the amounts earned by the mother and by other members of the family, and as to other sources of income. The information obtained on these points was not considered, however, so reliable as that on the father's earnings. The earnings of mothers, for example, were de-

[1] Since the material is used simply to show the correlation between earnings and mortality, a uniform error of overstatement would not affect the validity of the conclusion.

rived largely from keeping lodgers and boarders, and when this was the source it was difficult to separate the net from the gross receipts. In determining total family income, moreover, not only were net receipts difficult to secure in cases, for example, of income from rents of houses owned but not lived in by the families, but also perplexing questions arose as to whether a given item, for example, money borrowed or money returned, constituted true income. Furthermore, difficulties in comparing incomes arose from the fact that some families may have invested their savings in the homes occupied by them and, though neither receiving nor paying rent, may have incurred various expenses connected with such ownership, while others may have invested their savings in such a way as to secure definite cash incomes. For the purposes of this study the amount which it would have been most desirable to know was the portion of the income spent for the subsistence and maintenance of the family or, better still, the portion actually expended for the infant's maintenance and in ways that would influence the infant's health, including expenditures for the health of the mother. Since, however, this amount could not be ascertained and even the total family income was subject to so much doubt, the data on father's earnings appeared to furnish the nearest approximation to a satisfactory measure of the effect of economic status on infant mortality. These data, therefore, have been used as the principal basis of classification, a procedure which seems, moreover, as will be shown later, to be justified by the correlation found between father's earnings, total income, and father's earnings per capita of members of the family.

All these data relating to amounts of earnings, it should be emphasized, were secured before the marked rise in prices and wages during and following the war, and since the time from the first to the last study of the series was relatively short—about three years—the figures for annual earnings in the different cities, so far as changes in the level of prices and wages is concerned, may be considered fairly comparable.

Classifying each case according to the earnings of the father during the year immediately following the birth (so far as these earnings were available for the family), Table I shows a marked relationship between infant mortality and father's earnings. The higher the earnings the smaller the proportion of deaths. For infants whose fathers earned less than $450 the rate was 166.9, as compared with only 59.1 for those whose fathers earned $1,250 or over. The group of babies whose fathers were classified as having "no earnings" had the highest rate of all, 210.9.

TABLE I

INFANT MORTALITY RATES, BY CAUSE OF DEATH AND EARNINGS OF FATHER: LIVE BIRTHS IN SEVEN CITIES

Earnings of father	Live births	Infant deaths	Infant mortality rates *						Neonatal mortality rate †
			All causes	Gastric and intestinal diseases	Respiratory diseases	Early infancy	Epidemic and other communicable diseases	All other causes	
Total	21,536	2,368	110.0	32.3	19.2	35.9	6.8	15.7	44.4
Under $450	3,085	515	166.9	60.6	33.4	43.8	12.3	16.9	55.8
$450 to $549	2,827	355	125.6	44.9	23.3	35.4	6.4	15.6	46.0
$550 to $649	2,908	339	116.6	36.5	18.9	37.5	9.3	14.4	43.3
$650 to $849	5,050	543	107.5	27.1	19.6	37.4	5.5	17.8	46.5
$850 to $1,049	3,345	277	82.8	19.7	13.5	30.2	4.2	15.2	38.0
$1,050 to $1,249	1,391	89	64.0	8.6	7.9	25.9	3.6	18.0	33.1
$1,250 and over	2,252	133	59.1	8.0	6.2	32.4	2.2	10.2	38.2
No earnings	313	66	210.9	86.3	47.9	41.5	16.0	19.2	60.7
Not reported	365	51	139.7	41.1	16.4	49.3	16.4	16.4	41.1

* Deaths under 1 year per 1,000 live births. † Deaths under 1 month per 1,000 live births.

The distribution of births in the several father's earnings groups is shown also in Table I. A striking fact was the small proportion, only 10.5 per cent, of the infants studied who were born in families in which the father earned more than $1,250. On the other hand, 41.0 per cent were born in families in which the father earned less than $650.

In practice the father's earnings, except perhaps in the comparatively small group in which they exceeded $1,250, constituted in the vast majority of cases by far the largest part of the family's income, and in fact may be said to have determined its economic status. In Baltimore, for example, only 7 per cent of the families had any income from insurance, investments, or rents from tenants outside their own dwellings; the proportion was greatest, as would be expected, for families which were classed in the group $1,250 and over on the basis of father's earnings.[1] In a considerable proportion, 23 per cent, of the families the earnings of the father were supplemented by those of the mother or of children, but the amounts received from these supplementary sources of income were small. In about half the cases the mother's gross earnings were less than $150. The amounts reported for the children were on an average higher; in about half the cases the total earnings of all the children in the family who worked amounted to $300 or over. If the classification had been based on family income instead of on father's earnings, about one fifth of the entire number of infants would have been placed in higher groups,

[1] See *Infant Mortality: Results of a Field Study in Baltimore, Md., based on births in one year*, by Anna Rochester, U. S. Children's Bureau publication No. 119, p. 35.

but the other four fifths would have been placed in the same groups by either method.

Classifying the births both by father's earnings and by total family income indicated, surprising as it may appear, that in families with equal amounts of father's earnings the infant mortality rate was higher when more was contributed to the income from other sources, in the majority of cases from the earnings of the mother or of older children. This is doubtless explained in part by the influence of the mother's employment in increasing infant mortality, and partly by such factors as high order of birth for infants in families which had children old enough to be employed.

The significance of father's earnings in terms of economic pressure is shown in Table II, which sets forth the correlation between the

TABLE II

PROPORTION OF BIRTHS IN FAMILIES HAVING PER CAPITA INCOME FROM FATHERS' EARNINGS OF LESS THAN $100, AND MEDIAN INCOME PER CAPITA, BY EARNINGS OF FATHER: LIVE BIRTHS IN SEVEN CITIES

Earnings of father	Live births			Median per capita income *
	Total	In families with per capita income * less than $100		
		Number	Per cent	
Total †	18,587	3,175	17.1	$177
Under $450	3,077	1,508	49.0	$101
$450 to $549	2,826	853	30.2	$129
$550 to $649	2,905	513	17.7	$161
$650 to $849	5,047	256	5.1	$234
$850 to $1,049	3,341	43	1.3	$265
$1,050 to $1,249	1,391	2	0.1	$338

* Income from father's earnings only.
† Excludes births in families in which fathers earned $1,250 and over, or had no earnings, and those in which earnings or number in family were not reported.

amount of these earnings and their distribution per capita of the members of the family. The estimated median per capita income from father's earnings increased from $101 for the group "under $450" to $338 for the group $1,050 to $1,249." [1]

Since the average number in the family was slightly greater in the lower than in the higher father's earnings groups, the variations in economic pressure appeared slightly greater than variations in the

[1] In this calculation the group $1,250 and over was not considered; for the rest the father's earnings in each family were assumed to be equal to the middle amount in the group in which it was classed, thus, $500 for families in the group "$450 to $549," $600 for those in the group "$550 to 649," etc., and the per capita income was found by dividing by the number as reported in each family. For the group "under $450" an arbitrary figure of $400 was set as the approximate average earnings for purposes of this calculation.

earnings themselves. In other words, as the father's earnings, which constituted the bulk of the family income, decreased the economic pressure increased at a slightly more rapid rate. In general, the lower the earnings the greater was the economic pressure as indicated by their distribution per capita.

Though the differences between father's earnings groups may, therefore, be taken as indicating differences in average per capita incomes or in average economic pressure, in each group were included some families that would have been placed in a relatively higher, and others that would have been placed in a relatively lower, position on a per capita income scale than on the father's earnings scale. If the families were all classified, therefore, on a per capita income scale, the true influence of economic pressure—or, if extreme, of poverty—as a factor in infant mortality would be more clearly presented.

In Table III the result of a classification of births by per capita amount of father's earnings is presented for all the infants born in families in which the father earned less than $1,250, that is, for those groups in which the father's earnings most closely represented family income. The relationship shown by this grouping between economic pressure and the infant mortality rate was even more striking—as indeed would be expected—than when no account was taken of the number of members in the family for whom the earnings had to provide. The conclusion seems justified, therefore, that an analysis based on father's earnings alone yields substantially the same results as an analysis based on father's earnings per capita of his household and gives, in fact, trustworthy evidence as to the relation of economic need to mortality among the infants studied. The following discussion, therefore, runs in terms of father's earnings, except where the number in the family has to be taken into consideration as one of the items of the analysis.

TABLE III

INFANT MORTALITY RATES, BY PER CAPITA INCOME FROM FATHER'S EARNINGS: LIVE BIRTHS IN SEVEN CITIES

Per capita income from father's earnings	Live births	Infant deaths	Infant mortality rate
Total *	18,587	2,117	113.9
Less than $50	227	49	215.9
$50, less than $100	2,948	418	141.8
$100, less than $200	6,900	850	123.2
$200, less than $400	8,000	769	96.1
$400 and over	512	31	60.5

* Excludes births in families in which the father earned $1,250, had no earnings, or earned amounts which were not reported, and a few cases in which the number in the family was not reported or in which the infant lived in an institution.

Not only did the general rates of infant mortality decrease as the earnings of the fathers increased, but, with one or two exceptions, the rates from the several causes, as shown in Table I, exhibited the same tendency. The mortality from gastric and intestinal diseases showed, perhaps, the most marked variation by earnings, the rate in the lowest earnings group, "under $450," being seven and one half times that in the highest, "$1,250 and over."

The mortality from respiratory diseases likewise showed a marked negative correlation with father's earnings. For the infants of fathers who earned less than $450 the rate was 33.4, as compared with 6.2, less than one fifth as high, for those of fathers who earned $1,250 and over. As would be expected, the mortality among the babies whose fathers were reported to have had "No earnings" was highest of all.

Similar correlations appeared for the other causes of death. From epidemic diseases the lowest mortality was found to have been in the father's earnings group "$1,250 and over," and the highest in the two groups "No earnings" and "Under $450." Likewise from causes peculiar to early infancy the rates for the infants of fathers who had the lowest earnings were higher than for those of fathers who earned $850 and over.

The correlations just discussed are the total correlations associated with differences in earnings. As already noted, they may arise from a causal relation between poverty and the death of babies or may constitute merely a result of the association of low earnings with greater prevalence of artificial feeding or with other factors that, though not themselves a result of insufficient income, appear to cause high infant mortality. Before concluding that the association of excessive mortality with low earnings proves that a high infant death rate is a direct result of poverty, the question must be answered—Is this correlation due to the influence of other causes? Three other causes or groups of causes should be considered in this connection; type of feeding, color and nationality of mother, and the group of factors associated with frequency of births.

Since the mortality among the artificially fed infants was found to have been three or four times as high as that among the breast fed,[1] the inquiry is pertinent in this connection—Was the high rate among infants in families in which father's earnings were low due to an unusual prevalence of artificial feeding in those families? The analysis shows that, on the contrary, the giving of exclusively artificial food was markedly less prevalent in the low than in the high income groups.

[1] See Woodbury, R. M. "The Relation Between Breast and Artificial Feeding and Infant Mortality." *The Am. Journal of Hygiene*, Vol. II. (Nov. 1922) pp. 668–687.

Of the months lived from birth up to the end of the ninth by babies whose fathers earned less than $450, 22.0 per cent were months of artificial feeding, as compared with 30.0 per cent of those lived by babies whose fathers earned $1,250 or over. Evidently, therefore, no part of the excess mortality among infants in the low income groups can be explained by any greater tendency toward artificial feeding in these groups.

Since infant mortality varied markedly in the different race and nationality groups the question should next be examined—Were the infants of the groups in which the mortality rates were high found in disproportionate numbers in the low income groups? The race and nationality groups in which the infant mortality rates were above average were the Portuguese with a rate of 200.3, the French Canadian with a rate of 171.3, the Polish with a rate of 157.2, the colored with a rate of 154.4, and the group of "other foreign" [1] nationalities with a rate of 129.6. These five groups included 32.3 per cent of all the infants. But the earnings group "Under $450" included 55.0 per cent of the infants of these five race groups, while the group "$1,250 and over" included only 10.0 per cent. This markedly disproportionate weighting of the low income groups with infants of races with higher than average infant mortality rates might account, therefore, for part of the tendency toward high mortality in the low income groups.

Nevertheless, that this correlation with nationalities which had high infant mortality rates accounted for but a relatively small part of the excess mortality associated with low earnings is apparent from Table IV. In each of the principal color and nativity groups, the native white, the foreign-born white, and the colored, the relation between low earnings and a high infant death rate was striking. And among the native white infants alone—a group uninfluenced by any disproportionate weighting with colored or with foreign-born nationalities—the relation between mortality and father's earnings was even more striking than in the entire group studied. Furthermore, the differences between the average mortality rates in the three race and nativity groups appeared to have resulted largely from the unfavorable weighting of the foreign-born white and the colored with disproportionate numbers of infants in the low earnings groups, for the differences between the rates for the infants of the foreign born and colored mothers were relatively slight when comparisons were made within groups where earnings were substantially equal. The high rate of 166.9 in the earnings group "Under $450," for example, was not in-

[1] Includes foreign-born nationalities except Italian, German and Jewish and those mentioned in the text.

creased by the inclusion of a disproportionate number of infants of colored mothers, since the rate for such infants in this particular earnings group was in fact not so high as that for the infants of native white mothers. The conclusion may fairly be drawn, therefore, that the relationship found between low earnings of fathers and high infant mortality was not due to, but was independent of, the correlation between low earnings and color and nativity.[1]

TABLE IV

INFANT MORTALITY RATES, BY COLOR AND NATIVITY OF MOTHER AND EARNINGS OF FATHER; LIVE BIRTHS IN SEVEN CITIES

Earnings of father	Infant mortality rates			
	Total	Native white	Foreign-born white	Colored *
Average	110.0	93.9	123.7	152.3
Under $450	166.9	170.0	167.1	162.7
$450 to $549	125.6	121.0	118.4	163.7
$550 to $649	116.6	110.8	121.8	122.8
$650 to $849	107.5	99.5	119.6	102.7
$850 to $1,049	82.8	76.4	94.9	.....
$1,050 to $1,249	64.0	62.6	68.4	.....
$1,250 and over	59.1	57.6	60.0	.....
No earnings	210.9	187.5	234.2	.....
Not reported	139.7	154.7	129.5	.....

* Not shown where base is less than 100.

The third factor or set of factors which must be considered as having a possible influence in producing the correlation between low earnings and high infant mortality is the group of factors associated with frequency of births, including age of mother, order of birth, interval since preceding birth, and interval before succeeding pregnancy.[2]

Low economic status was found to be correlated with each of the four factors associated with birth frequency. The low earnings groups included a disproportionate number of infants of orders fifth and later, of those born to mothers under 20 or to those 35 years of age or over, of those who followed preceding brothers or sisters after short intervals, and of those whose mothers became pregnant during their first year of life—all four divisions which had higher than average mortality rates. In no case, however, was there any very great difference between earnings groups in the proportions of births in these four divisions. The percentages of infants born to mothers under 20 or 35

[1] A separate analysis of the rates by nationality of the foreign-born group leads to a similar conclusion; namely, that the relationship between low earnings and high infant mortality rates was, on the average, practically as marked within each nationality group as in the group as a whole.

[2] No "birth rate" applicable and peculiar to each individual infant can be calculated. The influence of the "birth rate" as a causal factor must, therefore, be of composite character and the joint resultant of the factors, age of mother, order of birth, interval since preceding birth, and interval before succeeding pregnancy.

years of age or over varied only from 30.7 for those whose fathers were classified as having had no earnings to 20.1 for those whose fathers earned $850 to $1,049. For short interval births the percentages varied from 29.4 per cent in the group "Under $450" to 20.4 per cent in that "$1,250 and over." For infants whose mothers became pregnant again during their first year of life the proportions varied from 16.1 per cent in the group "Under $450" to 7.9 per cent in that "$1,250 and over." [1] The most marked variations were found between the percentages of births of orders fifth and later, which varied from 34.5 in the earnings group "Under $450" to only 18.1 in that $1,250 and over."

Not only were these differences comparatively small, but to a certain extent the four factors, order of birth, age of mother, interval since preceding birth and interval before a following pregnancy, were interrelated. Births of high orders were commonly to mothers of older ages, and were frequently associated with short intervals.

The problem of isolating the influence of these factors from that of low income over the infant mortality rate offers some difficulty. The most appropriate method of analysis is Westergaard's method of expected deaths, which is used in constructing Table V.[2] The first column shows for each earnings groups the ratio of actual deaths to deaths that would have been expected if the average rate of infant

TABLE V

RELATIVE INFANT MORTALITY BY EARNINGS OF FATHER, WHEN INFLUENCE OF AGE OF MOTHER AND ORDER OF BIRTH, AND WHEN INFLUENCE OF INTERVAL SINCE PRECEDING BIRTH, IS ELIMINATED

Earnings of father	Ratio of actual to expected deaths			
	Expected at average rates for single births; seven cities	Expected at rates for same order of birth and age of mother	Expected at average rates for all births; Baltimore	Expected at rates for same interval since preceding birth
Total	100.0	100.0	100.0	100.0
Under $450	152.8	142.5	151.4	140.3
$450 to $549	113.5	108.8	114.0	116.9
$550 to $649	103.0	101.1	105.1	100.5
$650 to $849	98.0	98.8	92.8	96.5
$850 to $1,049	75.4	78.1	69.1	66.1
$1,050 to $1,249	59.0	61.9	64.3	66.9
$1,250 and over	54.5	59.6	62.5	60.8
No earnings	200.0	191.6	200.7	211.4
Not reported	130.1	128.6	135.5	131.1

[1] In the "No earnings" group, including families in which the father died or deserted or was sick or unemployed during the entire year, very few, only 1.4 per cent, of the mothers became pregnant during the infant's first year.

[2] For a discussion of the method of analysis, see Woodbury, R. M. "Westergaard's method of expected deaths as applied to the study of infant mortality," QUARTERLY PUBLICATIONS OF THE AMERICAN STATISTICAL ASSOCIATION, (Sept. 1922) Vol. XVIII, pp. 366–376.

mortality for all births had prevailed in each earnings group, forming a series of ratios which states simply the total correlation of earnings with the infant mortality rate. The second column shows a series of ratios that expresses the relation between earnings and mortality after the influences of age of mother and order of birth have been eliminated. A comparison of the second with the first series of figures shows that the correlation between earnings of father and the infant mortality rate was reduced by the elimination of the influence of age of mother and order of birth combined, but that the reduction was relatively slight; the effect of eliminating the influence of these factors did not alter materially the marked correlation between low earnings of father and a high mortality rate. The next two columns of the table show the effect of eliminating interval since the preceding birth; in this case the series of ratios after the influence of interval has been eliminated should be compared with the series in the third column, which, however, shows a trend quite similar to that indicated in the first column. The effect of eliminating the influence of interval since preceding birth as a factor evidently did not change significantly the excess mortality associated with low income. The correlation between a high infant mortality rate and low income was still as marked as before.

Unfortunately the births for which interval was available included only about half those for which other data were obtained and the number was too limited to permit of eliminating jointly all three influences —age of mother and order of birth and interval since preceding birth. But since the net result of eliminating the last mentioned factor was practically not to alter the trend of the rates by father's earnings, and since the effect of eliminating the other two was slight, the conclusion may fairly be drawn that even after the eliminating of all three factors together there would still be found the same marked correlation between low income and high infant mortality.

No attempt was made to eliminate the effect of interval before succeeding pregnancy, since the analysis showed that the possible influence of this factor was limited to less than one twentieth of the months under one year of age lived by all the babies in the study. This influence was found to be due largely to the greater prevalence of artificial feeding among infants whose mothers had become pregnant again during their first year and, therefore, so far as the effect of this factor on the relation between economic status and infant mortality is concerned, it was offset by the smaller proportions of other babies of the same earnings groups who were fed artificially, with the net result already mentioned that fewer babies of the low than of the high earnings groups were given artificial food.

The evidence indicates, therefore, that the relation found to exist between low earnings and high infant mortality was not to be explained by correlations with type of feeding, with color and nationality of mother, or with factors associated with the frequency of birth, but must be considered to indicate some direct or indirect causal connection.

The influence of the father's earnings upon his infant's chances of life was undoubtedly indirect and was exerted through the relation of the family's economic status to the mother's health and care during pregnancy and confinement, and to the care of the baby during his first year. As already suggested, this influence may be found to be exerted through such intermediate factors as housing congestion or the mother's employment during pregnancy or during the infant's first year, which may be shown partially to account for the correlation between father's earnings and infant mortality. When the meager earnings of the father have to be supplemented by the mother's gainful labor outside the home, the lack of income which is responsible for the mother's employment is, fundamentally, responsible also for whatever increased mortality is found to be associated with such employment.

An intimate connection was found, as shown in Table VI, between father's earnings and the amount and quality of prenatal care in the one city, Baltimore, for which information on the latter points was

TABLE VI

AVAILABILITY OF MEDICAL AND NURSING CARE TO MOTHERS OF LEGITIMATE INFANTS DURING PREGNANCY AND CONFINEMENT, BY EARNINGS OF FATHER; BALTIMORE, 1915 *

Earnings of father	Per cent of mothers having no medical prenatal care	Per cent of mothers having prenatal care of Grade A †	Per cent of mothers not attended by physicians at confinement	Per cent of mothers having 10 or more visits from physician following delivery ‡	Per cent of mothers having no nursing care during confinement period	Per cent of mothers who stayed in bed or in hospital less than 7 days following delivery
Total.....	47.5	5.1	32.6	15.6	28.6	8.1
Less than $450...	53.0	1.3	35.7	8.9	37.2	12.8
$450 to $549.....	58.7	1.8	43.6	8.2	30.1	11.8
$550 to $649.....	59.8	1.6	43.7	9.2	29.9	
$650 to $849.....	52.7	3.5	36.6	14.5	30.4	7.1
$850 to $1,049 ...	40.5	6.6	27.9	20.7	26.0	4.2
$1,050 to $1,249..	30.0	8.2	18.1	24.1	25.0	3.4
$1,250 to $1,449..	32.9	9.2	19.1	26.9	19.8	2.4
$1,450 to $1,849..	21.3	16.4	8.5	40.1	15.1	1.7
$1,850 and over..	13.8	30.3	6.6	40.3	7.5	
No earnings.....	41.4	1.7	22.0	8.2	34.9	7.3
Not reported....	39.7	4.2	25.1	15.7	31.0	7.3

* These percentages are based upon mothers of legitimate births in Baltimore in 1915.

† To qualify as grade A care, the care received by the mother must have satisfied all four of the following requirements: (1) Monthly visits to clinic from 5th to 9th month of pregnancy or supervision by private physician from the 5th to the 9th montn; (2) monthly urinalysis from the 5th to 9th month; (3) at least an abdominal examination; (4) pelvic measurements, if a primipara.

‡ Exclusive of mothers with complications of confinement.

secured. Thus, 53 per cent of the mothers in the group "Under $450" had no medical prenatal care, as compared with only 13.8 per cent of those in the group "$1,850 and over." And of the mothers in the lowest earnings group 35.7 per cent were not attended by a physician at confinement, as compared with only 6.6 per cent of those in the highest. Similarly with regard to nursing care during the confinement period; 37.2 per cent of the mothers in the lowest, as compared with only 7.5 per cent of those in the highest, group had no such care. The proportion of mothers who stayed in bed or in a hospital for less than seven days following delivery varied from 12.8 per cent in the lowest to 1.7 per cent in the highest earnings group.

Though ignorance in the poorer families of the rules of health doubtless played some part in producing these differences, the obvious relation between lack of means and inability to purchase medical and nursing services and competent advice and to follow out that advice suggests that lack of means or economic pressure must have borne a heavy responsibility. The health program recommended by the best medical authorities for pregnant women includes wholesome and nourishing food, spending at least two hours daily in the open air, and freedom from worry and overwork. This program is obviously one which can be more easily and faithfully observed by women in comfortable circumstances than by those who have to struggle daily to make ends meet.

One item of considerable importance from the point of view of the mother's health and her observance of this health program is her employment away from home during pregnancy. A close relationship was found to exist between low earnings of the father and such employment. The proportion of mothers employed away from home during pregnancy was highest, 37.4 per cent, in the "No earnings" group, was 28.3 per cent in the group "Under $450," and was lowest, 1.9 per cent, in the group "$1,250 and over."

The influence upon infant mortality of employment of the mother during pregnancy is shown in Table VII. The infants of mothers who worked away from home had a markedly high rate of mortality from causes peculiar to early infancy, a fact which is shown not only in the specific rate from these causes but also in an excessively high neonatal mortality rate. Furthermore, the proportion of premature births was considerably higher than the proportions either for mothers employed at home or for mothers not employed. The infants of mothers who worked in gainful occupations at home, on the other hand, had a mortality rate from all causes slightly higher than that among the infants of mothers who did not work at all; but the rate from causes

peculiar to early infancy, the neonatal mortality rate, and the percentage of premature births were all below those for infants of mothers who were not employed. This somewhat unexpected result may be due in part to the fact that the amount and character of work done by the mothers who kept lodgers or boarders were not essentially different from those involved in performing ordinary household and family duties, and in part, perhaps, to a tendency on the part of mothers who were not in the best physical condition or who were in ill-health not to work at all. The group of mothers who were not at

TABLE VII

INFANT MORTALITY RATES FROM ALL CAUSES AND FROM CAUSES PECULIAR TO EARLY INFANCY, NEONATAL MORTALITY RATE, AND PER CENT OF PREMATURE BIRTHS, BY EMPLOYMENT OF MOTHER DURING PREGNANCY: LIVE BIRTHS IN EIGHT CITIES.

Employment of mother during pregnancy	Live births	Infant deaths	Infant mortality rate			Per cent of premature births
			From all causes	From causes peculiar to early infancy	In first month (neonatal rate)	
Total	22,967	2,555	111.2	36.1	44.8	5.0
Employed away from home	3,084	543	176.1	50.3	67.2	6.1
Employed at home	3,788	434	114.6	27.2	33.6	3.5
Not employed	16,089	1,576	98.0	35.6	43.2	5.2
Not reported	6	2	.....	....	....	..

work may therefore have been somewhat unfavorably weighted by mothers who would have been in one of the other groups if they had enjoyed normal health. That the infants of mothers who were employed away from home during pregnancy showed higher mortality rates than those of mothers who were not so employed or were not employed at all is, in any case, significant evidence of the influence of this type of mother's employment.

Specific evidence in regard to the correlation between poverty and the quality and quantity of care which the baby received during his first year was difficult to secure, and was limited to a few points, such as feeding, housing congestion, and the gainful employment of the mother, especially away from home.

A principal element of infant care, maternal nursing during the first nine months, has already been discussed; artificial feeding was less practiced in the lower than in the higher earnings groups. As to the quality of artificial food used by the several groups, specific evidence unfortunately is lacking. The analysis indicated, however, that artificial feeding was fraught with greater dangers to the baby's

health in the lower earnings groups. Doubtless such feeding as practiced in the families with larger incomes was accompanied by safeguards that were not employed to the same extent in the poorer families. These safeguards probably included the use of pure milk, the practice of proper sterilization of bottles and nipples, the advice of a competent physician in regard to a suitable milk modification formula and his supervision over the feeding of the baby, and the purchase and correct utilization of the ingredients and medicines prescribed. Each of these safeguards, however, involves the expenditure, if not of money, at least of the mother's time; and therefore their use, other things being equal, would naturally be intimately associated with the income available for the care of the infant.

That differences in the safeguards surrounding the practice of artificial feeding did not explain fully, however, the greater mortality among the infants whose fathers' earnings were comparatively low is shown by the fact that a contrast similar to that between the rates for the artificially fed was also found between those for the breast fed babies in the several earnings groups. Other causes of fewer deaths among the babies whose fathers had the higher earnings doubtless included the greater availability of nursing and medical services. In such families, too, the mothers probably were freer from household cares, either because of the employment of servants or because of the use of mechanical aids to work, and as a consequence had more time to care for their babies.

A striking correlation was found between low income and housing congestion. To test this relation the deaths of infants under two weeks of age were omitted, since it is doubtful whether housing congestion could have been a factor in these deaths. Of the infants who survived for at least two weeks and whose fathers earned less than $450, 72.2 per cent, as compared with only 17.5 per cent of those whose fathers earned $1,250 or over, were found in families which lived one or more persons per room. The significance of this correlation becomes evident when the rates of mortality are considered; in families which lived less than one person per room the infant death rate was only 52.1, as compared with 94.9 in families which lived one but less than two persons per room and with 135.7 in families which lived two or more persons per room. Since housing congestion is one of the consequences following in the train of low earnings, the latter must bear the responsibility for the high mortality caused by the former.

Markedly greater proportions of mothers were employed during their infants' first year of life in the low than in the high income groups. The proportion of mothers employed away from home was highest,

38.0 per cent, in the "No earnings" group, was 19.6 per cent in the group "Under $450," and was lowest, 0.6 per cent, in the two groups "$1,050 to $1,249" and $1,250 and over." As in the case of employment during pregnancy, the higher earnings groups had very few mothers employed away from home. But in each group a much smaller proportion of mothers was found to have been employed away from home during the infant's first year than was found to have been so employed during pregnancy. Nevertheless, the mother's employment away from home during the baby's first year of life was found to have been associated with a mortality 1.6 times the average rate for babies in the same father's earnings groups.

To summarize, in this group of legitimate city births a marked correlation was found between low earnings of father and high infant mortality. Low earnings of the father were found to correspond to high economic pressure as measured by the per capita amount of earnings available for each member of the family. This relationship between earnings and mortality was found to be independent of type of feeding, of race and nationality, and of the factors involved in frequency of births. The conclusion may be drawn, therefore, that economic pressure due to low father's earnings is a causal factor in the infant death rate.

The influence of economic position is undoubtedly indirect. The study of the chain of causation connecting poverty with high infant mortality showed that the responsibility of low earnings extended over the excessive mortality associated with housing congestion and with the mother's employment both during pregnancy and during her child's first year of life. It showed also that a direct relation existed between lack of means and the kind of care available both for the mother during pregnancy and confinement and for the baby during his first year. The analysis suggests, therefore, that lack of care is another intermediate factor.

The presence of these intermediate factors in the chain of causation, though it does not relieve the economic factor of its present responsibility for high infant mortality, suggests that the chain may be broken through measures that will make the intermediate factors independent of the amounts of money available for family expenditures. Such measures as the free provision of adequate prenatal and confinement care for mothers and of medical supervision over infants through municipal health centers or clinics, if available to all without reference to their economic position, would tend, if thoroughly carried out, not only to reduce the infant mortality rate but also to lessen, if not to eliminate, its dependence upon economic position. Similar results,

moreover, would be anticipated from the prohibition of the work of mothers during those periods when it has the greatest adverse influence upon infant mortality, together with such special subsidies as would make such work unnecessary. This dependence is to be regarded not as an immutable verity but as a consequence of the fact that at present the health and care of infants depend upon the resources available in each family and that, therefore, particular conditions that may in a given case reduce or destroy such resources may place the infant's life in jeopardy.

SEASONAL VARIATION AS A RELATIVE OF SECULAR TREND[1]

By Lincoln W. Hall, *University of Pennsylvania*

The method of monthly or quarterly means for the calculation of the seasonal variation of any economic time series introduces the question of whether the residuals, obtained by subtracting the ordinates of secular trend from the original data, shall be used directly as absolute[2] quantities, or whether they shall first be expressed as relatives of the secular trend and then used in the calculation of seasonal variation. It is the purpose here to examine into the difference between the two alternatives and to present some reasons for the use of the residuals as a relative of secular trend so that the seasonal variation would also be a relative of secular trend.

Let $x_0, x_1, \ldots x_k$ represent the original data and

$u_0, u_1, \ldots u_k$ represent the corresponding ordinates of secular trend.

Then the residuals, r, can be expressed as $x_0-u_0, x_1-u_1, \ldots x_k-u_k$ so that $r_i=x_i-u_i$ $(i=o, \ldots k)$.

Then considering quarterly data[3] the absolute seasonal variation, v, is

$$v_0=\frac{(x_0-u_0)+(x_4-u_4)+\ \ldots\ +(x_{4(n-1)}-u_{4(n-1)})}{n}$$

$$v_1=\frac{(x_1-u_1)+(x_5-u_5)+\ \ldots\ +(x_{1+4(n-1)}-u_{1+4(n-1)})}{n}$$

$$v_2=\frac{(x_2-u_2)+(x_6-u_6)+\ \ldots\ +(x_{2+4(n-1)}-u_{2+4(n-1)})}{n}$$

$$v_3=\frac{(x_3-u_3)+(x_7-u_7)+\ \ldots\ +(x_{3+4(n-1)}-u_{3+4(n-1)})}{n}$$

where n=number of years.

Since the v's after v_3 repeat in the order of v_0, v_1, v_2, v_3, the absolute cyclical fluctuation, w becomes

$$w_i=x_i-u_i-v_j \begin{Bmatrix} i=0 & \ldots & k \\ j=0 & \ldots & 3 \end{Bmatrix}$$

$$\begin{aligned} w_0&=x_0-u_0-v_0 \\ w_1&=x_1-u_1-v_1 \\ w_2&=x_2-u_2-v_2 \\ w_3&=x_3-u_3-v_3 \end{aligned} \qquad (1)$$

[1] Received by the Editor, December 22, 1923.

[2] The word absolute is not used here strictly in its mathematical sense, but only to differentiate between a quantity and a percentage.

[3] The argument holds for monthly data.

$w_4 = x_4 - u_4 - v_0$

. . . . . .

. . . . . .

$w_k = x_k - u_k - v_3$ (where the k^{th} term $= 4n-1$ (n integral)).

In order to express the seasonal variation as a relative of secular trend, the residuals, r, must be first expressed as a relative of secular trend. The r's then become $\frac{x_i - u_i}{u_i}$ $(i = 0 \quad . \quad . \quad . \quad k)$.

Then the seasonal variation, v^1, becomes

$$v_0^1 = \frac{\left(\frac{x_0-u_0}{u_0}\right)+\left(\frac{x_4-u_4}{u_4}\right)+ \quad . \quad . \quad . \quad +\left(\frac{x_{4(n-1)}-u_{4(n-1)}}{u_{4(n-1)}}\right)}{n}$$

$$v_1^1 = \frac{\left(\frac{x_1-u_1}{u_1}\right)+\left(\frac{x_5-u_5}{u_5}\right)+ \quad . \quad . \quad . \quad +\left(\frac{x_{1+4(n-1)}-u_{1+4(n-1)}}{u_{1+4(n-1)}}\right)}{n}$$

$$v_2^1 = \frac{\left(\frac{x_2-u_2}{u_2}\right)+\left(\frac{x_6-u_6}{u_6}\right)+ \quad . \quad . \quad . \quad +\left(\frac{x_{2+4(n-1)}-u_{2+4(n-1)}}{u_{2+4(n-1)}}\right)}{n}$$

$$v_3^1 = \frac{\left(\frac{x_3-u_3}{u_3}\right)+\left(\frac{x_7-u_7}{u_7}\right)+ \quad . \quad . \quad . \quad +\left(\frac{x_{3+4(n-1)}-u_{3+4(n-1)}}{u_{3+4(n-1)}}\right)}{n}$$

Then the absolute cyclical fluctuation, w^1, as a relative of secular trend when calculated from a relative seasonal variation, or $\frac{w_i^1}{u_i}$ $(i = 0 \quad . \quad . \quad . \quad k)$ is

$$\frac{w_0^1}{u_0} = \frac{x_0-u_0}{u_0} - v_0^1 \qquad \frac{w_i^1}{u_i} = \frac{x_i-u_i}{u_i} - v_j^1 \begin{cases} i = 0 \quad . \quad . \quad . \quad k \\ j = 0 \quad . \quad . \quad . \quad 3 \end{cases}$$

$$\frac{w_1^1}{u_1} = \frac{x_1-u_1}{u_1} - v_1^1$$

$$\frac{w_2^1}{u_2} = \frac{x_2-u_2}{u_2} - v_2^1$$

$$\frac{w_3^1}{u_3} = \frac{x_3-u_3}{u_3} - v_3^1$$

$$\frac{w_4^1}{u_4} = \frac{x_4-u_4}{u_4} - v_0^1$$

. . . . . .

. . . . . .

$$\frac{w_k^1}{u_k} = \frac{x_k-u_k}{u_k} - v_3^1 \text{ (where } k^{\text{th}} \text{ term} = 4n-1 \text{ } (n \text{ integral})\text{)}.$$

Multiplying by the respective u in the above expressions in order to secure the absolute cycle w^1, this becomes

$$
\begin{aligned}
w_0^1 &= x_0 - u_0 - u_0 v_0^1 \\
w_1^1 &= x_1 - u_1 - u_1 v_1^1 \\
w_2^1 &= x_2 - u_2 - u_2 v_2^1 \\
w_3^1 &= x_3 - u_3 - u_3 v_3^1 \\
w_4^1 &= x_4 - u_4 - u_4 v_0^1 \\
&\cdots\cdots\cdots\cdots \\
&\cdots\cdots\cdots\cdots
\end{aligned}
\qquad w_i^1 = x_i - u_i - u_i v_j^1 \begin{cases} i=0 \ . \ . \ . \ k \\ j=0 \ . \ . \ . \ 3 \end{cases} \tag{2}
$$

$w_k^1 = x_k - u_k - u_k v_3^1$ (where k fulfills the same condition as above)

It is seen that (1) and (2) represent the absolute cycles w and w^1 obtained in the first case by calculating an absolute seasonal variation and then subtracting this variation from the absolute residual in order to get the absolute cycle; and in the second case calculating the residuals as a relative of secular trend, obtaining the seasonal variation as a relative of secular trend from the relative residuals, subtracting the relative seasonal from the relative residual to obtain a relative cycle, and then calculating this relative cycle as an absolute cycle.

It is evident that similarity or difference between the two absolute cycles calculated by the two methods can be seen by comparing expressions (1) and (2). From (1) and (2) it can be seen that the difference between the two cycles depends entirely upon the difference between v_j and $u_i v_j^1$. This becomes the difference between

$$\frac{(x_0-u_0)+(x_4-u_4)+ \ . \ . \ . \ +(x_{4(n-1)}-u_{4(n-1)})}{n} \tag{3}$$

and

$$u_p \frac{\left[\left(\frac{x_0-u_0}{u_0}\right)+\left(\frac{x_4-u_4}{u_4}\right)+ \ . \ . \ . \ +\left(\frac{x_{4(n-1)}-u_{4(n-1)}}{u_{4(n-1)}}\right)\right]}{n} \tag{4}$$

Where (3) and (4) represent the difference for the first quarter of each year.

$$p=(0,\ 4,\ 8 \ . \ . \ . \ 4(n-1)).$$

Evidently only one quarter of the year need be considered because what is true of this quarter is true of the other three quarters.

Now (3) becomes

$$\frac{(x_0+x_4+x_8+ \ . \ . \ . \ +x_{4(n-1)})}{n} - \frac{(u_0+u_4+u_8+ \ . \ . \ . \ +u_{4(n-1)})}{n} \tag{5}$$

And (4) becomes

$$(6) \qquad u_p\left[\frac{\left(\frac{x_0}{u_0}+\frac{x_4}{u_4}+\frac{x_8}{u_8}+\ \ .\ \ .\ \ .\ \ +\frac{x_{4(n-1)}}{u_{4(n-1)}}\right)}{n}-1\right]$$

It will be seen that the *only* condition under which (5) and (6) can become identical is that of no secular trend;[1] in which case

$$u_0=u_1=u_2=u_p=\ \ .\ \ .\ \ .\ \ =u_k=0.$$

Under this condition (5) and (6) both reduce to

$$\frac{(x_0+x_4+x_8+\ \ .\ \ .\ \ .\ \ +x_{4(n-1)})}{n}-u.$$

Where u is the constant of the equation of secular trend and is the arithmetic mean of the series.

Therefore in all instances except where there is no secular trend, (eliminating remote accidental instances), w and w^1 will be different absolute cycles, and they may or may not be different by a sufficient amount to give rise to a different interpretation of the economic events which they represent.

The same results can be obtained by comparing the relative cycles $\frac{w_i}{u_i}$ and $\frac{w_i^1}{u_i}$. The difference between the relative cycles will depend upon the difference between $\frac{v_j}{u_i}$ and v_j^1 and it will be seen that they also become identical only when there is no secular trend.

A fundamental property of the seasonal variation as a relative of secular trend can be seen by imposing the condition that the residuals are increasing or decreasing at a constant ratio to their respective secular trend. Then $\frac{x_i-u_i}{u_i}=\pm c$ or $x_i-u_i=\pm c\,u_i$.

Then the absolute seasonal variation v or

$$\frac{\Sigma(x_p-u_p)}{n}\ (p=0,\ 4,\ 8,\ \ .\ \ .\ \ .\ \ 4\,(n-1)$$

or

$$(p=2,\ 6,\ 10\ \ .\ \ .\ \ .\ \ 2+4\,(n-1))$$

or

$$(p=1,\ 5,\ 9\ \ .\ \ .\ \ .\ \ 1+4\ (n-1))$$

or

$$(p=3,\ 7,\ 11\ \ .\ \ .\ \ .\ \ 3+4\ (n-1))$$

becomes $\frac{c\Sigma u_p}{n}$ which is $c\,Mu_p$ where Mu_p is the arithmetic mean of the

[1] If the secular trend has the form $y=mx+b$, then $m=0$ and $b=a$, or in general all the coefficients of the independent variable are zero.

u_p. This means that the absolute seasonal variation remains unchanged although the magnitude of the series may have changed greatly.

The seasonal variation as a relative of secular trend v^1 or $\frac{\Sigma\left(\frac{x_p - u_p}{u_p}\right)}{n}$ becomes under the above condition c, the constant ratio, and when expressed as an absolute it becomes $c\,u_p$, where p is the same as above.

This analysis shows the fundamental difference between the two methods of calculating the seasonal variation. When it is calculated directly as an absolute, the seasonal variation does not change during the period under observation even though the magnitude of the series is changing rapidly. This means that at the beginning of the period the seasonal variation will probably be too large while at the end of the period it will probably be too small, if the magnitude of the series is increasing.

When the seasonal variation is calculated as a relative of secular trend first and then expressed as an absolute, it is seen that the amount of variation is changing with and in proportion to the secular trend, so that the only constant is the constant ratio of secular trend.

This is as it should be, because unless the natures of the seasonal variation and cyclical fluctuation are themselves changing, their absolute amounts should be changing in proportion to the change in the magnitude of the series, as measured by the secular trend. Therefore when only the magnitude of a series is changing the ratio of the seasonal variation to the secular trend should be constant while its absolute amount should be variable.

Because of the above defect, when the seasonal variation is calculated directly as an absolute, it is necessary when a change in secular trend occurs—particularly a large change—to recalculate the seasonal variation for the period following the change in trend.[1] When the seasonal variation is calculated as a relative of secular trend, no such recalculation is necessary no matter how much the secular trend may change.

When using the seasonal variation as a relative of secular trend for the purpose of calculating the cyclical fluctuations, the following problem arises:

(A) Shall the relative seasonal be subtracted from the relative residual in order to get the relative (of secular trend) cycle and then turn this relative cycle into an absolute cycle, or—

[1] See the writer's *A Study of the Cyclical Fluctuations Occurring in the National Bank System during the Years 1903 to 1921*, Ch. 2, where a change of secular trend occurred in the banking data at the beginning of 1915.

(B) Shall the relative seasonal be turned into an absolute seasonal, and then this absolute seasonal subtracted from the absolute residual to get the absolute cycle, and then change the absolute cycle into a relative cycle?

The specific questions involved under (A) and (B) are, (1) will (A) and (B) give the same absolute cycle; (2) the same relative cycle; and (3) will both (A) and (B) give absolute cycles which, when added to the absolute seasonal and secular, will result in the original data. It is evident that if (A) and (B) do not give the same results, then that method which will yield result (3) is the method which must be used.

According to (A) the relative cycle $\frac{w_i^1}{u_i}$ becomes

$$\frac{x_i-u_i-u_iv_j^1}{u_i} \tag{7}$$

And the absolute cycle is

$$\frac{u_i(x_i-u_i-u_iv_j^1)}{u_i}=x_i-u_i-u_iv_j^1 \tag{8}$$

Since the absolute seasonal is $u_iv_j^1$, the absolute secular, absolute seasonal, and absolute cyclical when added give

$$u_i+u_iv_j^1+(x_i-u_i-u_iv_j^1)=x_i \tag{9}$$

Therefore (A) meets the test of yielding components which when added together will give the original data from which the components were obtained.

According to (B) since the absolute seasonal is $u_iv_j^1$ the absolute cyclical becomes

$$x_i-u_i-u_iv_j^1 \tag{10}$$

Dividing by the respective secular trend the relative cycle becomes

$$\frac{x_i-u_i-u_iv_j^1}{u_i} \tag{11}$$

And adding the absolute secular, absolute seasonal, and absolute cyclical, we have

$$u_i+u_iv_j^1+(x_i-u_i-u_iv_j^1)=x_i \tag{12}$$

By comparing (8) and (10); (7) and (11); and (9) and (12), it is seen that (A) and (B) may be used interchangeably. If it is desired to obtain only the relative cycle, the absolute cycle need not be computed, and if it is desired to obtain only the absolute cycle the relative cycle need not be computed. It is evident that if the relative cycle can be obtained without computing the absolute cycle that considerable labor

of computation is saved, which may become an important consideration when a number of series are being studied. The advantage of being able to compute the absolute cycle without computing the relative cycle depends on the fact that when a straight line secular trend is used the absolute seasonal is considerably easier to compute than the absolute cyclical. The formula for computing the absolute seasonal under the above condition is

$$(13) \qquad u_i v_j^1 = v_j^1 u_j + 4(n-1) v_j^1 M \quad \begin{Bmatrix} j=0, 1, 2, 3 \\ i=0, \ . \ . \ . \ k \end{Bmatrix}$$

Where n = number of years and M = the slope of the straight line of trend.

For monthly data the multiplier $4(n-1)$ becomes $12(n-1)$.

Formula (13) means that after the seasons of the first year have been calculated by taking percentages of the secular trend, that the same seasons for all succeeding years (during which the same straight line is used as secular trend) can be calculated by the simple process of successive addition or subtraction instead of taking a separate percentage of secular trend for each season of each year.

It has been shown that the seasonal variation as a relative of secular trend is unaffected by changes in trend and this property makes this method of calculating seasonal variation superior for the purposes of current computation of cycles. A moving seasonal variation could be constructed so as to discover actual changes in the nature of the seasonal variation of any series. For instance, the relative residuals for the preceding three cycles could be used, or possibly for the preceding ten years, and this structure moved up regularly so that the seasonal variation would be currently recalculated as new items were obtained.

It seems evident that the theorems of Professor Hart[1] will apply equally well to the calculation of seasonal variation from relative residuals as from absolute residuals. If his data are thought of as consisting of relative residuals instead of absolute residuals, his theorems will hold for relative seasonal variation. For this reason in the case of the use of a moving seasonal variation, the time covered by the periods of the three (or other number) preceding cycles rather than some arbitrary number of years would seem better adapted to such a moving structure.

The absolute seasonal variation calculated directly from the absolute residuals is not well adapted to a moving structure because of its liability to distortion by changes in the secular trend.

For purposes of illustration a series was calculated by both methods.

[1] Journal of the American Statistical Association, September 1922.

The series chosen was "overdrafts" of all national banks as given by the Comptroller's reports during the years 1903 to 1921.[1] The data were arranged and used as quarterly data.

Table I contains, in addition to the original data and secular trend, the calculations resulting from the use of the seasonal variation directly as an absolute. Table II contains the results obtained by using the seasonal variation as a relative of secular trend.

TABLE I

OVERDRAFTS, 1903–14

1 Original data (000) omitted	2 Secular trend	3 Residuals from secular trend (absolute)	4 Seasonal variation (absolute)	5 Cyclical fluctuation (absolute)	6 Cyclical fluctuation % (relative of secular trend)
35721	41748	− 6027	+2092	− 8119	−19.4
27258	41318	−14060	−6951	− 7109	−17.2
27191	40887	−13696	−4879	− 8817	−21.3
51399	40457	+10942	+9740	+ 1202	+ 2.9
42401	40026	+ 2375	+2092	+ 283	+ .7
26800	39595	−12795	−6951	− 5844	−14.7
31777	39165	− 7388	−4879	− 2509	− 6.4
54941	38734	+16207	+9740	+ 6467	+16.6
43749	38304	+ 5445	+2092	+ 3353	+ 8.7
30367	37873	− 7506	−6951	− 555	− 1.4
29905	37442	− 7537	−4879	− 2658	− 8.7
54473	37012	+17461	+9740	+ 7721	+20.8
47256	36581	+10675	+2092	+ 8583	+23.4
30034	36151	− 6117	−6951	+ 834	+ 2.3
32475	35720	− 3245	−4879	+ 1634	+ 4.5
53735	35289	+18446	+9740	+ 8706	+24.7
41916	34859	+ 7057	+2092	+ 4965	+14.2
32870	34428	− 1558	−6951	+ 5393	+15.6
30443	33998	− 3555	−4879	+ 1324	+ 3.8
37544	33567	+ 3977	+9740	− 5763	− 7.1
29625	33137	− 3512	+2092	− 5604	−16.9
23335	32706	− 9371	−6951	− 2420	− 7.0
24705	32275	− 7570	−4879	− 2691	− 8.3
38910	31845	+ 7075	+9740	− 2665	− 8.3
29077	31414	− 2337	+2092	− 4429	−14.0
24584	30984	− 6400	−6951	+ 551	+ 1.7
29552	30553	− 1001	−4879	+ 3878	+12.6
41907	30122	+11785	+9740	+ 2045	+ 6.7
34027	29692	+ 4335	+2092	+ 2243	+ 7.5
25743	29261	− 3518	−6951	+ 3433	+11.7
29541	28831	+ 710	−4879	+ 5589	+19.3
47066	28400	+18666	+9740	+ 8926	+31.4
40507	27969	+12538	+2092	+10446	+37.3
23397	27539	− 4142	−6951	+ 2809	+10.2
27150	27108	+ 42	−4879	+ 4921	+18.1
35950	26678	+ 9272	+9740	− 468	− 1.7
23982	26247	− 2265	+2092	− 4357	−16.6
19849	25816	− 5967	−6951	+ 984	+ 3.8
20168	25386	− 5218	−4879	− 339	− 1.3
26493	24955	+ 1538	+9740	− 8202	−32.8
22307	24525	− 2218	+2092	− 4310	−17.5
19006	24094	− 5088	−6951	+ 1863	− 7.6
18377	23664	− 5287	−4879	− 408	− 1.7
27460	23233	+ 4227	+9740	− 5513	−23.7
21838	22802	− 964	+2092	− 3056	−13.4
15485	22372	− 6887	−6951	+ 64	+ .4
17142	21941	− 4799	−4879	+ 80	+ .3
18797	21511	− 2714	+9740	−12454	−57.8

$y = -430.6x + 41748.$

[1] Data taken from the writer's *A Study of the Cyclical Fluctuations Occurring in the National Bank System during the Years 1903 to 1921.*

TABLE I (*continued*)

OVERDRAFTS, 1915–21

1 Original data (000) omitted	2 Secular trend	3 Residuals from secular trend (absolute)	4 Seasonal variation (absolute)	5 Cyclical fluctuation (absolute)	6 Cyclical fluctuation % (relative of secular trend)
7047	6080	+ 967	− 226	+ 1193	+19.6
5905	6535	− 630	−1685	+ 1055	+16.1
5060	6991	− 1931	− 767	− 1164	−16.6
7211	7447	− 236	+1475	− 1711	−22.9
5493	7902	− 2409	− 226	− 2183	−27.6
6994	8358	− 1364	−1685	+ 321	+ 3.8
7839	8814	− 975	− 767	− 208	− 2.3
9317	9269	+ 48	+1475	− 1427	−15.3
7666	9725	− 2059	− 226	− 1833	−18.8
8069	10181	− 2112	−1685	− 427	− 4.1
9607	10636	− 1029	− 767	− 262	− 2.4
15044	11092	+ 3952	+1475	+ 2477	+22.3
13586	11547	+ 2039	− 226	+ 2265	+19.6
11662	12003	− 341	−1685	+ 1344	+11.1
14306	12459	+ 1847	− 767	+ 2614	+20.9
16814	12914	+ 3900	+1475	+ 2425	+18.7
13881	13370	+ 511	− 226	+ 777	+ 5.7
12421	13826	− 1405	−1685	+ 280	+ 2.0
15131	14281	+ 850	− 767	+ 1617	+11.1
23116	14737	+ 8379	+1475	+ 6904	+47.9
19215	15193	+ 4022	− 226	+ 4288	+24.9
16406	15648	+ 758	−1685	+ 2443	+15.6
17545	16104	+ 1441	− 767	+ 2208	+13.7
19277	16559	+ 2718	+1475	+ 1243	+ 7.5
12360	17015	− 4655	− 226	− 4389	−25.7
10770	17471	− 6701	−1685	− 5016	−28.7
12355	17926	− 5571	− 767	− 4804	−26.7
9949	18382	− 8433	+1475	− 9908	−53.9

$y = 455.6x + 6080.$

Since two secular trends were used, one from 1903 to 1914 and the other from 1915 to 1921, the seasonal variation as a relative was calculated separately for 1915 to 1921 so as to correspond to and compare with the separate calculation for this period of the absolute seasonal variation from absolute residuals. An inspection of the tables will readily show the differences resulting from the two methods.

In order to show the possible effect on the absolute seasonal calculation and thence on the cyclical fluctuation of a change in secular trend, the seasonal variation was calculated by both methods for the entire period 1903 to 1921. The results were, absolute seasonal, +1238, −5011, −3364, +6695; relative seasonal +3.1%, −18.5%, −12.1%, +23.4%. For the first quarter of 1915, the relative residual is +15.9%, and when the relative seasonal is subtracted from it, the result is +12.8% as the relative cyclical. For the same time the absolute residual is +967, and when the absolute seasonal of +1238 is subtracted algebraically from it, the result is −271, which when expressed as a relative of the secular trend becomes −4.4%. Therefore

the figures of relative cyclical to be compared are +12.8% and −4.4%. The difference between these figures seems rather great and might easily have led to serious error in the interpretation of the economic phenomena represented.

TABLE II

OVERDRAFTS, 1903–14

1 Residuals as relative of secular trend Per cent	2 Seasonal variation as a relative of secular trend Per cent	3 Seasonal variation as an absolute	4 Cyclical fluctuation as a relative of secular trend Per cent	5 Cyclical fluctuation as an absolute
−14.4	+ 6.2	+ 2588	−20.6	− 8600
−34.0	−21.6	− 8924	−12.4	− 5123
−33.5	−15.1	− 6174	−18.4	− 7523
+27.0	+29.5	+11935	− 2.5	− 1011
+ 5.9	+ 6.2	+ 2482	− .3	− 12
−32.3	−21.6	− 8553	−10.7	− 4237
−18.8	−15.1	− 5914	− 3.7	− 1449
+41.8	+29.5	+11427	+12.3	+ 4764
+14.2	+ 6.2	+ 2375	+ 8.0	+ 3064
−19.8	−21.6	− 8180	+ 1.8	+ 682
−20.1	−15.1	− 5654	− 5.0	− 1872
+47.2	+29.5	+10919	+17.7	+ 6551
+29.2	+ 6.2	+ 2268	+23.0	+ 8414
−16.9	−21.6	− 7808	+ 4.7	+ 1699
− 9.1	−15.1	− 5394	+ 6.0	+ 2143
+52.3	+29.5	+10411	+22.8	+ 8046
+20.2	+ 6.2	+ 2161	+14.0	+ 4880
− 4.5	−21.6	− 7436	+17.1	+ 5887
−10.5	−15.1	− 5134	+ 4.6	+ 1564
+11.8	+29.5	+ 9903	−17.7	− 5941
−10.6	+ 6.2	+ 2054	−16.8	− 5567
−28.7	−21.6	− 7064	− 7.1	− 2322
−23.4	−15.1	− 4874	− 8.3	− 2679
+22.2	+29.5	+ 9395	− 7.3	− 2325
− 7.4	+ 6.2	+ 1947	−13.6	− 4272
−20.7	−21.6	− 6692	+ .9	+ 279
− 3.3	−15.1	− 4614	+11.8	+ 3605
+39.1	+29.5	+ 8887	+ 9.6	+ 2892
+14.6	+ 6.2	+ 1840	+ 8.4	+ 2494
−12.0	−21.6	− 6320	+ 9.6	+ 2809
+ 2.5	−15.1	− 4354	+17.6	+ 5074
+65.7	+29.5	+ 8379	+36.2	+10281
+44.8	+ 6.2	+ 1733	+38.6	+10796
−15.0	−21.6	− 5948	+ 6.6	+ 1818
+ .2	−15.1	− 4094	+15.3	+ 4148
+34.8	+29.5	+ 7871	+ 5.3	+ 1414
− 8.6	+ 6.2	+ 1626	−14.8	− 3885
−23.1	−21.6	− 5576	− 1.5	− 387
−20.6	−15.1	− 3834	− 5.5	− 1396
+ 6.2	+29.5	+ 7363	−23.3	− 5815
− 9.0	+ 6.2	+ 1519	−15.2	− 3728
−21.1	−21.6	− 5204	+ .5	+ 120
−22.3	−15.1	− 3574	− 7.2	− 1704
+18.2	+29.5	+ 6755	−11.3	− 2625
− 4.2	+ 6.2	+ 1412	−10.4	− 2371
−30.8	−21.6	− 4832	− 9.2	− 2058
−21.9	−15.1	− 3314	− 6.8	− 1492
−12.6	+29.5	+ 6247	−42.1	− 9056

TABLE II (*continued*)
OVERDRAFTS, 1915–21

1 Residuals as relative of secular trend Per cent	2 Seasonal variation as a relative of secular trend Per cent	3 Seasonal variation as an absolute	4 Cyclical fluctuation as a relative of secular trend Per cent	5 Cyclical fluctuation as an absolute
+15.9	− 2.2	− 134	+18.1	+ 1100
− 9.6	−13.3	− 869	+ 3.7	+ 242
−27.6	− 7.1	− 496	−20.5	− 1433
− 3.2	+12.9	+ 961	−16.1	− 1199
−30.5	− 2.2	− 174	−28.3	− 2236
−16.3	−13.3	− 1111	− 3.0	− 251
−11.1	− 7.1	− 625	− 4.0	− 353
+ .5	+12.9	+ 1196	−12.4	− 1149
−21.2	− 2.2	− 214	−19.0	− 1848
−20.7	−13.3	− 1354	− 7.4	− 753
− 9.7	− 7.1	− 755	− 2.6	− 277
+35.6	+12.9	+ 1431	+22.7	+ 2518
+17.7	− 2.2	− 254	+19.9	+ 2298
− 2.9	−13.3	− 1596	+10.4	+ 1248
+14.8	− 7.1	− 884	+21.9	+ 2729
+30.2	+12.9	+ 1666	+17.3	+ 2234
+ 3.8	− 2.2	− 294	+ 6.0	+ 802
−10.2	−13.3	− 1838	+ 3.1	+ 429
+ 5.9	− 7.1	− 1013	+13.0	+ 1857
+56.8	+12.9	+ 1901	+43.9	+ 6470
+26.5	− 2.2	− 334	+28.7	+ 4360
+ 4.8	−13.3	− 2081	+18.1	+ 2832
+ 9.0	− 7.1	− 1143	+16.1	+ 2593
+16.4	+12.9	+ 2136	+ 3.5	+ 580
−27.4	− 2.2	− 374	−25.2	− 4288
−38.4	−13.3	− 2323	−25.1	− 4385
−31.1	− 7.1	− 1272	−24.0	− 4302
−45.9	+12.9	+ 2371	−58.8	−10809

THE MEASUREMENT OF SEASONAL VARIATION [1]

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I

The determination of seasonal variation in an economic series is rendered difficult by the fact that such series are usually subject to *three other main types of fluctuation.* We are assuming that the changes in any economic series may be classified under four heads as suggested by Professor W. M. Persons.[2] The four components of the total fluctuation are: seasonal variation, secular trend, cyclical fluctuation, and irregular deviations. *Seasonal variation* is that part of the fluctuation due to the persistent tendency for certain months of each year to be regularly higher than certain other months of the year, irrespective of the disturbing influence of cyclical fluctuation and irregular deviations. The amount of seasonal variation, for a particular month, is measured by an *index of seasonal variation.* This index is the per cent ratio which tends, on the average over a considerable time interval, to hold between the actual value and the normal value in that month. Thus, if there were no cyclical fluctuation and no irregular items, this ratio would hold exactly between the actual value for the month and the normal value (due to secular trend alone). *Secular trend* is the long-time tendency of the items of the series to grow or decline regardless of the temporary seasonal and cyclical and irregular movements. This tendency is generally assumed to be constant in amount over a considerable interval of years. *Cyclical fluctuation* is the wave-like variation which reflects the succession of prosperity and depression in business conditions. This movement is not periodic in a mathematical sense; the *cycles* are of varying duration in time, have differing maximum deviations from the normal, and vary widely in shape. *Irregular deviations* are the residual elements of change not included in the three other components of the total fluctuation. They are usually abrupt in occurrence, of short duration, and of large magnitude; and they do not occur regularly at any particular time of the year or at any specific stage of the cycle. These deviations affect some economic series very notably, and have almost no part in the fluctuation of certain other series.

[1] Received by the Editor, January 14, 1924.
Professor W. L. Crum of Harvard University examined the original draft of this paper and made many valuable suggestions.

[2] *Rev. Econ. Stat.*, Prel. Vol. I, p. 8.

The statistical data which constitute an economic record are subject therefore to three types of fluctuation other than seasonal variation, and most of these component fluctuations are not strictly uniform in length or size or shape. The serious obstacle to the measurement of seasonal variation rests in the fact that each actual item is affected in an unknown manner and to an unknown extent by the other influences. The statistical scheme used for measuring seasonal movement should be designed to eliminate, so far as possible, the effects of these other component fluctuations and to present an index which reflects seasonal variation alone.

II

Among the numerous devices which have been suggested for the purpose there are three to which we shall give especial consideration. It is the object of the present section to discuss them, in the light of the above description of the peculiar difficulties of the problem.

1. One of the simplest schemes for determining seasonal variation is the so-called method of *monthly means*.[1] According to this plan the seasonal movement is indicated by twelve averages: the average of all the January items, the average of all the February items, and so on up to the average of all the December items. The resulting numbers may be expressed in percentage form by dividing each of them by the mean of the twelve. The chief advantage of the method is the extreme ease of computation, but this is offset by the failure of the scheme to make adequate allowance for the effects of the three other types of fluctuation.

The allowance which the method makes for cyclical fluctuation consists in covering a sufficiently long time interval to warrant the expectation that cyclical influences will tend to cancel each other. No exact cancellation, however, can be relied upon.

The method admits of an extension, developed by Professor G. R. Davies (*loc. cit.*, p. 119), which eliminates the effect of secular trend. When secular trend is not eliminated its influence is likely to be apparent as in the case of the December to January drop in the seasonal curves of many of Professor Kemmerer's charts (for example, Chart IV, *loc. cit.*, p. 24).

The method makes no provision for the influence of irregular deviations, and it is here that we find the chief objection to its use. The arithmetic average is peculiarly subject to extreme items, and it is for that reason that a monthly seasonal index obtained by this method

[1] Recently discussed (Jour. Amer. Stat. Ass'n, Sept., 1922) by W. L. Hart; but already widely used, notably by E. W. Kemmerer in *Seasonal variations in the relative demand for money and capital in the U. S.;* and presented and illustrated by G. R. Davies in his *Economic Statistics.*

may be governed more by an exceptional irregular deviation than by the systematic seasonal movement. Moreover, even if there are no clearly irregular items, the average for any particular month may not be truly typical of that month. Thus, the average of nine January items (assuming a series with a nine year total interval) may not be a good measure of the statistical array composed of those items, unless they are fairly well clustered about their average. The method, as generally used, affords no device for testing whether the individual monthly means are truly typical.

Professor Kemmerer has attempted (*loc. cit.*, p. 13) to eliminate the effects of the irregular items by the use of indexes, but it is doubtful if this device is successful. The arbitrary selection of the highest and lowest items (for any one month, say January) as the 100 and 0 of his per cent scale makes *all* the indexes of that month depend on these possibly irregular extreme items, and it amounts to selecting as a basis of comparison that most unstable of dispersion measures, the range.

2. The essence of the *median-link-relative* method[1] is as follows: the calculation of month-to-month link relatives from the original data, the verification of the existence and estimate of the general nature of the seasonal movement by use of a multiple frequency table, the selection of the median of the link relatives for each month, the adjustment of these medians to allow for trend and other discrepancies, and the derivation of the final seasonal index expressed as a percentage of the average for the year. The scheme does not have ease of application to commend it, as does the method of monthly means.

The method contemplates no direct allowance for the effects of cyclical fluctuation, but requires merely that the interval covered by the series be sufficiently long to insure that the individual medians be satisfactorily typical. There is an indirect tendency to minimize the influence of cycles because of the comparison, in the link-relative operation, of each month with that immediately preceding. This fact tends in general to free the link-relatives from cyclical disturbances.

The first adjustment, the logarithmic correction, of the system aims to remove the effects of secular trend. This adjustment is occasionally quite different from its theoretical value, estimated from trend alone, and contains therefore other elements of discrepancy than the trend; but the introduction of the adjustment appears in most cases to furnish an adequate correction of the seasonal index for secular trend.

The use of the median, rather than the arithmetic mean, is primarily

[1] Originated by Prof. W. M. Persons, in *Rev. Econ. Stat.*, Prel. Vol. I; and discussed and reconsidered in various articles: W. L. Crum in Jour. Amer. Stat. Ass'n for March, 1923; A. Fisher, *ibid.*, June, 1923; W. M. Persons, *ibid.*, June, 1923; E. B. Wilson, *ibid.*, September, 1923.

for the purpose of avoiding the effects of the irregular items. These items are likely to appear, in the table of link relatives, as extremely high or extremely low ratios; and the value of the median will be independent of these extreme ratios. Prof. W. L. Crum (*loc. cit.*, p. 614) maintains that the frequency distribution in an economic series will often have such a form that the median is more stable than the mean, and suggests a slight modification of the median in cases in which the central concentration is not well defined. Mr. Arne Fisher (*loc. cit.*, p. 793) and Professor E. B. Wilson (*loc. cit.*, p. 841) have discussed this question further, giving respectively unfavorable and favorable views of the median.

The median-link-relative method appears to make a satisfactory allowance for the disturbances due to the components, other than seasonal, of the fluctuation; and, except for the lengthy computations involved in its application, it appears superior to other methods now in use.

3. A third method is that of *moving averages*. The scheme in its original form, in which the elimination of seasonal variation is attempted by replacing the actual item for a particular month by the average of the twelve monthly items of which it is the center (with an adjustment to insure exact centering), is now seldom used. What it really assumes is that any one interval of twelve months is as good a picture of the seasonal swing as any other. It makes no direct allowance for any of the three disturbing influences, but indirectly makes some allowance for all of them. If the secular trend is a straight line the method makes adequate provision for that influence. If there are no irregular deviations (within the twelve-months interval), the tacit assumption is made that the cyclical element in a particular monthly item is identical with the average cyclical fluctuation over the corresponding twelve months. This is scarcely true in general, although it may frequently happen to be true. If, however, the twelve-months interval *does* contain one or more irregular deviations (none of which occurs in the month under examination) the above assumption requires a modification: the cyclical element in the particular month is assumed identical with the twelve-months average of cyclical fluctuations *and* irregular deviations. If an irregular deviation occurs in the month in question, a corresponding different modification is necessary. The modified forms of the assumption, which the very general occurrence of irregular deviations necessitates, are obviously objectionable; and the method is not a satisfactory device for eliminating seasonal variation.

There has recently been developed a method which makes use of the

moving average, but is not directly dependent on the same assumptions.[1] Instead of using the median of link-relatives, this method uses the median of ratios between the actual items for any one month (say all the Januaries) and the values for that month estimated by a twelve-months moving average. The resulting medians may be converted to percentages and used as indexes of seasonal variation. The scheme makes allowances for disturbing influences, similar to those made in the median-link-relative method; but it involves even more laborious computation.

III

The present section describes a scheme for measuring seasonal variation which makes allowance for the effects of other types of fluctuation without involving excessively laborious computations. In fact, the method consists in computations, after the elimination of secular trend from the original data (an operation generally essential in the analysis of time date, regardless of the process used in getting seasonal indexes), scarcely more extensive than those of the monthly mean device. The plan assumes that the original data have been corrected for secular trend by getting the per cent ratio of each actual item to the corresponding ordinate of trend, and then provides for: the formation of a multiple frequency table of these ratios, in order to verify the existence and obtain an estimate of the nature of the seasonal movement; the selection, from each monthly distribution of ratios, of a typical value, which is usually the mean of a middle group of items; the adjustment of the twelve typical ratios, which are crude measures of seasonal variation, so that their average for the year is 100 per cent.

The method has been tested by application to the six series described in Table I; and, in order that the steps of the process may be clearly understood, the details of the calculation for the first series (a, Bank Clearings of New York City) are given herewith. It is noted that the ratios of the actual items to the ordinates of trend are obtained in order to eliminate trend from the original data, whatever device is used for determining the seasonal index; and the computation of these ratios is not therefore part of the labor of deriving seasonal indexes by this scheme. It should be remarked that, although each of the six illustrative series has a straight line trend fitted by the method of Professor Persons,[2] ratios of actual items to ordinates of trend might equally well be based upon a curvilinear trend if such were appropriate

[1] Presented by D. F. Jordan in *Business Forecasting;* and quite widely used, for instance by the Federal Reserve Bank of New York.

[2] *Rev. Econ. Stat.*, Prel. Vol. I, p. 12

TABLE I

THE SERIES TESTED AND THE ELEMENTS OF THEIR RESPECTIVE SECULAR TRENDS FOR THE INTERVAL 1903-1913

Symbol	Name of series	Source of data *	Monthly increment	Average (for July 1, 1908)
a	Bank Clearings of New York City	64	1.7	748.6
b	Tonnage of Pig Iron Produced in the United States	66	7.94	1950.9
c	Bank Clearings of the United States Outside New York City	68	2.26	491.8
d	Values of Building Permits Issued for Twenty Leading Cities	74	1.20	408.9
e	Rate of Interest on 60-90 Day Commercial Paper in New York	99	−.0016	4.895
f	Dividend Payments by Industrial Corporations	164	1.35	273.9

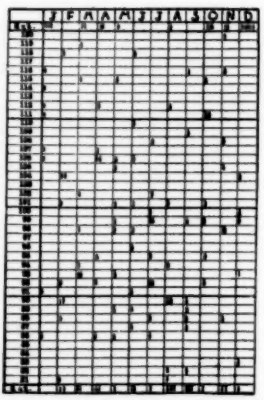
* The page number, in *Rev. Econ. Stat.*, Prel. Vol. I, where the original monthly items are given. The month-to-month link relatives used in this study, February, 1903, to January, 1914, will be found on the same or adjacent pages of that volume.

to the series; the present method merely requires that the proper form of trend be used in finding the ratios.

The next step is the construction of the multiple frequency table, consisting of twelve adjacent columns, each giving the frequency of the ratios for a particular month in the year; an dthe result is shown in Table II. Although the concentration in any one column of this table is not close, the distributions in pairs of adjacent columns are on the whole displaced considerably relative to each other. This evident relative displacement, although not as satisfactory a confirmation of

TABLE II*

FREQUENCY DISTRIBUTIONS, MONTHLY, OF RATIOS OF ACTUAL ITEMS TO THE RESPECTIVE ORDINATES OF SECULAR TREND, FOR BANK CLEARINGS OF NEW YORK CITY



* In Tables II, V, VI, VII, VIII the left-hand marginal number is the lower limit of the one per cent class interval, for each row of the table.

seasonal swing as would be close concentration within each column, is taken as a verification of seasonal movement.

Table III gives crude and adjusted indexes for groupings of varying width. The crude indexes are obtained directly from Table II: the first set consists of the medians of the columns in Table II, the second set is formed from the means of the groups of three middle items in each column, and so on for the other four sets. The adjusted indexes are obtained from the crude, in each case, by dividing the twelve crude indexes by their average and reducing the results to a per cent basis. The adjusted indexes are therefore the final values giving the seasonal variation; and it remains only to determine that one of the six sets which is to be accepted.

Examination of the crude indexes in Table III shows that in April, August, October, and November the median differs by several points from the average of the three middle ratios. These discrepancies emphasize the lack of concentration in the columns of Table II, and show therefore that the median is not a precise measure of the typical ratio. On the other hand, there is a tendency, particularly noticeable in January and August and November, for the group average to drift as groups of increasing width are taken. This considerable change between the index for a narrow group (of three items) and the index for a broader group (of nine or eleven items) reflects the influence of the irregular extreme items. It is particularly apparent in the case of January that the indexes for the nine-item and eleven-item groups are drawn upwards by the extremely high ratios of 1910 and 1906.

TABLE III

CRUDE * AND ADJUSTED * INDEXES OF SEASONAL VARIATION FOR NEW YORK BANK CLEARINGS, BASED UPON MIDDLE GROUPS OF VARYING NUMBERS OF ITEMS, FOR EACH OF THE SEVERAL MONTHLY DISTRIBUTIONS

Number of items in group	Crude indexes						Adjusted indexes					
	1	3	5	7	9	11	1	3	5	7	9	11
January...	112	112	112	114	115	117	113	113	113	115	116	117
February..	90	91	92	92	91	91	90	92	93	93	91	91
March....	98	98	100	102	102	102	98	99	101	102	102	102
April.....	106	102	101	101	100	100	107	103	102	101	100	100
May.....	99	99	99	99	99	99	99	100	100	99	99	99
June.....	96	95	95	95	95	94	96	96	96	96	95	95
July.....	95	95	95	95	95	94	95	96	96	95	95	95
August....	90	87	87	88	89	90	90	88	88	88	89	91
September.	88	88	87	88	89	89	88	89	88	88	89	90
October...	111	108	107	108	108	108	112	109	108	108	109	108
November .	105	104	105	105	103	102	106	105	106	105	104	103
December..	104	109	110	110	109	108	105	110	111	110	110	109

* The *crude* indexes are the averages of the groups of ratios, and the *adjusted* indexes are the results of revising the crude indexes so that their annual average is 100 per cent.

In view of these observations, it seems unwise to base the index on a group of less than three items or more than seven items. In a particular problem the multiple frequency table (Table II) will usually give the clue: if the ratios are closely concentrated the middle three items are sufficient; if the scatter is quite considerable, five items may be required; and in rare cases seven items will furnish the most typical index. Since the labor of calculating indexes for several groupings is very slight, doubtful problems may be handled by trial. If, as in the present case, the changes in the indexes from the three-item column to the five-item column are less significant than those from the five-item to the seven-item column, the results of the five-item or the three-item computation may be selected as final. When it is decided which grouping to use, the crude indexes are found, and the adjusted indexes based upon them are taken as measuring seasonal variation.

It is obvious that this scheme of finding seasonal indexes makes adequate allowance for secular trend, for the trend is actually eliminated from the original data before the computation of the seasonal variation. Moreover, it provides against the influence of the irregular deviations by using a group of middle items, in any one month, from which the irregular extremes are omitted.[1] The provision against the effects of cyclical fluctuation consists chiefly in the insistence upon a series covering a long time interval, but the proper selection of the grouping used in getting the crude indexes tends to minimize the disturbing effects of non-uniformity in the cycles.

Viewed by itself, this scheme appears to have the great advantage of ease in computation, which is claimed for the method of monthly means, and to retain also those safeguards against the influence of the other components of total fluctuation, which are characteristic features

[1] Since the development of the analysis presented above, the method used by Dr. Lincoln W. Hall in *A Study of the Cyclical Fluctuations Occuring in the National Banking System during the Years 1903 to 1921* has been noted and examined. On page 22 of that work, he states under the third heading of his "summary of the various steps" that his "seasonal variation is obtained by taking an arithmetic mean of the residuals for each season." He presents on the succeeding pages the data and computations for loans and discounts for the two intervals 1903–1914 and 1915–1921. In this analysis he has allocated the dates of call on a quarterly basis, and treats each such time interval as a regular seasonal segment of the year. The seasonal variations are found for the two intervals as: −19, 30, 32, −16; and −5, −143, −108, 11, respectively (attempt to recompute these average residuals yields identical results in all cases except the first item of the earlier interval, which appears to be −36 instead of −19). To facilitate examination, these average residuals are thrown on a percentage basis by dividing them by the corresponding ordinates of trend for an annual segment at the middle of each of the two intervals, and increasing the resulting quotients by 100 per cent. Expressed in this manner, the indexes of seasonal variation according to Dr. Hall's process are: 99.2, 100.6, 100.6, 99.6; and 100.5, 99.2, 99.5, 100.8, respectively.

It may be suggested that Dr. Hall's method would be considerably improved by using per cent ratios of the actual items to the trend, instead of the actual residuals from trend. If we first convert each of his residuals to a per cent basis by dividing it by the corresponding ordinate of trend, and then take arithmetic means of the per cent ratios for each of the four seasons, we get as the seasonal indexes: 99.2, 100.5, 100.6, 99.6; and 100.5, 99.0, 99.5, 101.0, respectively, for the two intervals 1903–1914 and 1915–1921. The differences between these results and those obtained by the direct application of his

of the median-link-relative method. It remains, however, to subject the scheme to varying practical tests to determine the extent of its reliability and to reveal what limitations must be placed upon its use.

IV

To test the method, it was applied to each of the series described in Table I, and the results appear in the columns headed RTO (ratio-to-ordinate) of Table IV. To facilitate comparison the corresponding

TABLE IV

COMPARISON OF THE INDEXES OF SEASONAL VARIATION FOR THE SIX TEST SERIES AS OBTAINED BY THE *MLR* AND *RTO* METHODS, USING IN EACH CASE THE MEAN, OF THE MIDDLE THREE ITEMS IN EACH MONTHLY DISTRIBUTION, INSTEAD OF THE MEDIAN

Series	a		b		c		d		e		f	
Method	MLR	RTO	MLR	RTO	MLR	RTO	MLR	RTO	MLR	RTO	MLR	RTO
January...	115	113	100	98	110	108	68	70	97	99	128	128
February..	91	92	93	93	92	91	70	72	73	95	77	75
March....	101	99	105	106	102	102	117	123	96	98	117	113
April.....	102	103	102	106	99	100	139	133	93	91	121	114
May.....	101	100	103	109	97	99	124	117	91	87	70	71
June.....	96	96	97	103	96	98	122	121	90	86	93	98
July.....	95	96	95	100	98	99	107	106	93	95	129	133
August....	90	88	98	103	91	92	101	105	101	100	68	69
September.	89	89	98	101	93	93	95	95	109	108	92	90
October...	107	109	104	101	108	108	94	93	113	111	127	127
November.	102	105	100	90	105	105	87	86	111	114	71	71
December..	108	110	101	90	108	105	76	80	114	113	109	112

indexes obtained by the link relative method, the average of three middle links being used instead of the median, are shown in columns headed MLR. A graphic comparison is afforded by Charts A–F.

The explanation will be based upon a comparison of cases *b* and *c;* and Tables V, VI, VII, VIII furnish the material which is needed. The ratios for any one month are much more closely concentrated in Table V than in Table VI, and it is therefore to be expected that case

method are scarcely significant in this case; but in general it is to be expected that the ratio scheme would be the more satisfactory, particularly in all cases in which the trend is fairly steep.

If now the indexes for the two intervals are obtained by the ratio-to-ordinate method, using the averages of the middle six ratios in the 1903–1914 interval and the averages of the middle three ratios in the 1915–1921 interval, the resulting indexes are: 99.2, 100.5, 100.7, 99.6; and 99.2, 98.0, 99.8, 103.0, respectively. The differences in the earlier interval are slight, and indeed there the examination of the multiple frequency table would probably have led to the conclusion that the determination of the seasonal indexes could not be precise in this case. The differences in the later interval are quite marked, and here an examination of the multiple frequency table reveals the reason for the discrepancies. Dr. Hall's method yields indexes which are controlled largely by the extreme items, and the few very large residuals in the later interval are really decisive in determining his indexes. It is this blind use of the arithmetic average which is exceedingly dangerous; and it is the purpose of the ratio-to-ordinate method developed in the present paper, with its presentation of the ratios in the multiple frequency table and its elimination of the extreme deviations, to guard against these very dangers.

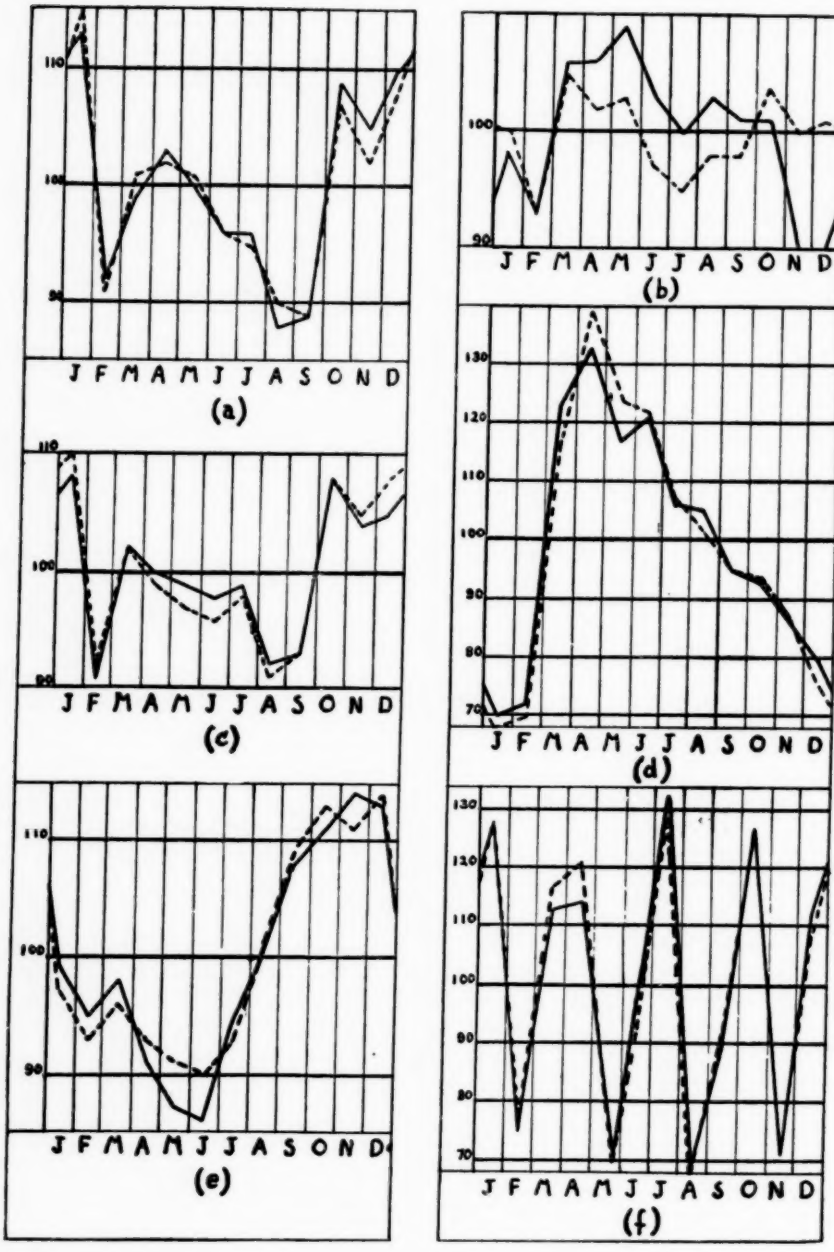

110
100
90
J F M A M J J A S O N D
(a)
100
90
J F M A M J J A S O N D
(b)
110
100
90
J F M A M J J A S O N D
(c)
130
120
110
100
90
80
70
J F M A M J J A S O N D
(d)
110
100
90
J F M A M J J A S O N D
(e)
130
120
110
100
90
80
70
J F M A M J J A S O N D
(f)

TABLE V

FREQUENCY DISTRIBUTIONS, MONTHLY, OF RATIOS OF ACTUAL ITEMS TO THE RESPECTIVE ORDINATES OF SECULAR TREND, FOR OUTSIDE BANK CLEARINGS

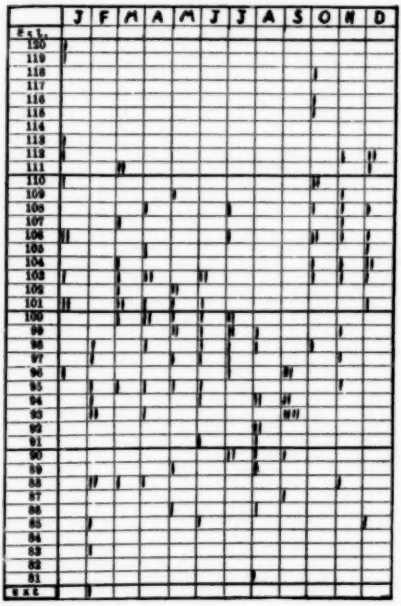
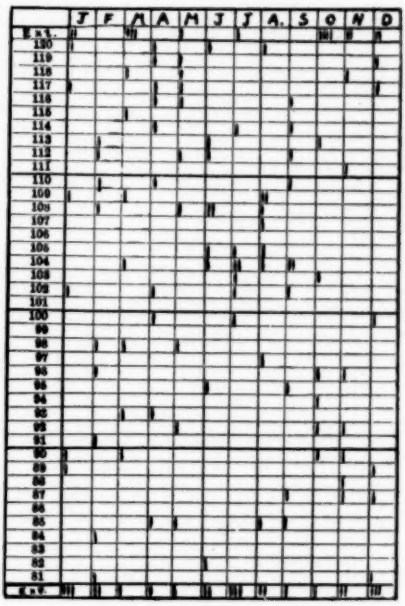


TABLE VI

FREQUENCY DISTRIBUTIONS, MONTHLY, OF RATIOS OF ACTUAL ITEMS TO THE RESPECTIVE ORDINATES OF SECULAR TREND, FOR PIG IRON PRODUCTION



c will afford a more precise seasonal index than case *b*. Moreover, examination of Tables VII and VIII indicates that the seasonal indexes obtained by the MLR method are probably more reliable for series *c* than for *b*. Comparative study of Tables VI and VIII suggests that the MLR index is probably somewhat more typical than the RTO index, but there is no sure conclusion. Neither method yields a highly precise index in a case in which the frequency tables show such great dispersion. It is true that Table VI has hardly greater dispersion than Table II (considered above), but the essential difference between them is that the relative displacement between adjacent columns is more clearly apparent in Table II than in Table VI. The conclusion appears then that a multiple frequency table showing the wide dispersion and the slight relative displacement between adjacent columns, which are the striking features of Table VI, is an indication that the RTO method does not yield a satisfactory seasonal index. A subsequent trial of the MLR method may show that the particular series does admit of a seasonal correction by that scheme; but, unless the corresponding frequency table is better formed than Table VIII,

TABLE VII

FREQUENCY DISTRIBUTIONS, MONTHLY, OF THE MONTH-TO-MONTH LINK RELATIVES, FOR OUTSIDE BANK CLEARINGS

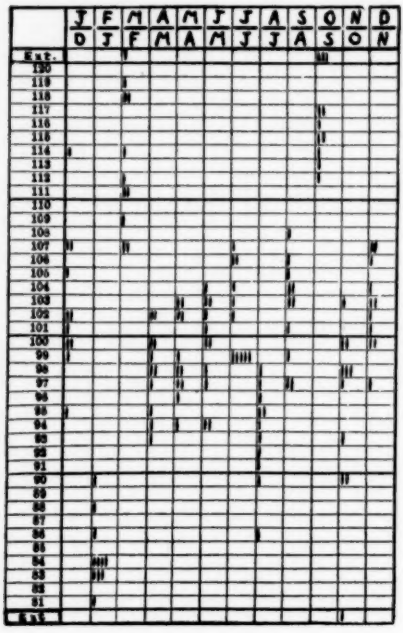
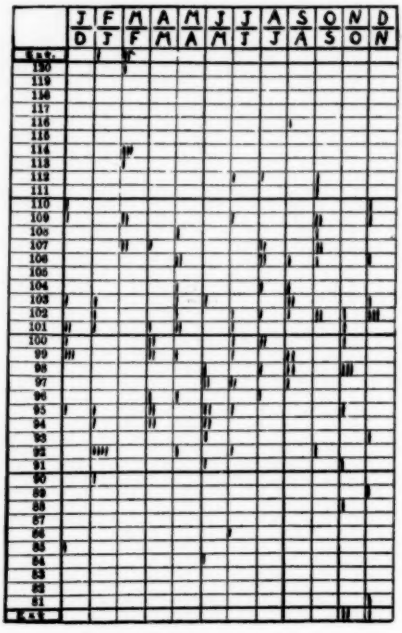


TABLE VIII

FREQUENCY DISTRIBUTIONS, MONTHLY, OF THE MONTH-TO-MONTH LINK RELATIVES, FOR PIG IRON PRODUCTION



even the results of this method must be accepted as merely approximate measures of the seasonal swing.

Investigation of the frequency tables, according to both methods, for series *e* confirm the above finding. In this case also the considerable discrepancy between the two results is explicable in the light of the wide dispersion in the frequency tables; and here also the MLR results are probably the better, although even they are not highly precise.

The general results of this study are as follows. The RTO method is designed to remove the influence of other-than-seasonal fluctuations in calculating seasonal indexes, and is nevertheless exceedingly simple and brief in application. The method can be used in cases in which the multiple frequency table has high concentration within each column (series *c* and *f* above), and in cases in which concentration within each column is slight but relative displacement between adjacent columns is well marked (series *a* and *d* above). It should not be used in those doubtful cases in which the concentration is poor and the relative displacement slight (series *b* above), and may be used only as

an approximation in borderline cases (series *e* above). In all the doubtful cases, however, it is well to try also the MLR method; sometimes its results will be more precise than those of the RTO scheme, and sometimes they will not. The very large number of practical problems in economic analysis for which the method is satisfactory justifies the attempt to use it in all cases. In the few cases in which it must be abandoned the only labor lost is that of constructing the multiple frequency table, and in the many cases to which it is applicable it represents a great economy of effort.

THE PROBLEM OF A STANDARD INDEX NUMBER FORMULA

By George R. Davies

Professor Fisher, in his recent work on *The Making of Index Numbers*, has shown that none of the index formulae that have been proposed meet perfectly all the tests to which they may be put. In other words, the finding of an index number is a matter of approximation rather than of exact mathematical procedure, even though the data may be complete and accurate. But he has also shown that there are a number of formulae which give fairly consistent results, and in which the degree of error may be considered almost or quite negligible. Out of these formulae he has picked one which he calls the ideal and which he uses as a theoretical standard of accuracy. This ideal formula meets all the mathematical tests that can be applied to it except the so-called circular test, and even this failure is a matter of only a small marginal error which can be plausibly excused.

It may, however, be argued that if any formula is to be taken as a theoretical standard it should be derived by a theoretically exact process, in accordance with ordinary mathematical procedure. It should, of course, measure up to the tests of accuracy usually applied. These tests would include the requirement that like factors should be treated alike, so that no difference in the result could appear from interchanging the order or altering the arrangement of these like factors. Professor Fisher thinks that this test should extend to the complete interchange of quantity and price factors, a point which will be discussed later. More fundamental is the test requiring that analogous index numbers of prices and quantities, when multiplied, should check with the actual change in the values produced or traded; if, for example, prices during a certain period rose to an index of 120 while quantities rose to an index of 110, then the product of these index numbers, or 132, should check with the actual increase in values concerned. The tests made would also include the time reversal test, which requires that in any given comparison the results should check when taken from alternate bases; for example, if prices in 1918 were found to be twice those in 1914, or 200 per cent, then prices in 1914 should prove to be half those of 1918, or 50 per cent. This time reversal test may also be extended to analogous triangular and circular tests, which would require that the results of more comprehensive comparisons should

check on various bases. Professor Fisher does not consider the triangular or circular tests essential, though he takes pride in showing that his ideal formula does in fact nearly conform to their requirements.

The point of view here taken is that the circular test, in which the triangular may be included, should be strictly applicable to an ideal index number formula. The circular test may be considered merely a complex form of the time reversal test, resting upon the same logical considerations. The arguments that may be urged for or against the one may obviously be applied to the other also. Thus Professor Fisher's argument that in respect to the price level city A might surpass city B, and city B might surpass city C, while at the same time city C surpasses city A, may be applied also to two cities or to two years. His argument rests on the fact that the commodities dealt with in each city differ so much that the various comparisons may naturally be inconsistent. But so also as between cities A and B; on the basis of commodities dealt in chiefly in A, prices there may be higher than in B, while at the same time on the basis of commodities dealt in chiefly in B, prices there may also be higher than in A. By the same criteria prices in 1913 might be higher than in 1914, and yet when computed from the basis of 1914, the latter prices might appear to be higher. Or at least the two results might fail to check as to the degree of change. This is in fact what actually occurs when the simpler formulae in common use are applied from different bases. But Professor Fisher takes the pains to develop a formula that takes into account the chief commodities marketed in both cities or years, so as to make the results consistent on reversed bases. If such consistency is desirable, as doubtless it is, it should also be desirable for a triangular comparison, or a more extensive one. That is, the circular test should apply. It should be possible to compute for cities A, B, and C, or for years 1914–1918 inclusive, a set of index numbers expressing the ratios of prices in the given cities or years, in so far as such a comparison can be made. Of course, the validity of any such comparison will obviously be limited by the degree of similarity of the goods dealt with in each place or year. If the goods are entirely different, no price comparison can be made. But in so far as multiple comparisons can be made, they should be consistent.

From the foregoing statement it can be seen that the term "base" may be used in two quite distinct senses, as Professor Fisher has so clearly shown. As most commonly used the term refers to the period taken as 100 per cent in a series of index ratios. If the given index ratios accurately express the quantities or prices in question, it is evident

that by a simple calculation the base as thus understood may be changed from one item to another. Thus if 100 and 200 correctly represent the ratio of the price levels in 1914 and 1918, respectively, then 50 and 100 will also correctly serve the same purpose, since they express the same ratio as before. And the same principle applies to index numbers of several items. The base is simply the item in a series of comparisons chosen for convenience to represent unity, or 100 per cent.

But in a more fundamental sense the base may refer to the time interval, or place, chosen to furnish the basis of comparison. As we have seen, on the basis of commodities used in city A, a price comparison with city B will vary from what it would have been if the commodities used in city B are taken as the basis of comparison. It is generally agreed, however, that a twofold comparison should involve a basis taking both cities equally into account, so that the results expressed in percentages of either A or B may check. It is logical to require also that similarly the basis of comparison of a group of several cities or of a series of years should be inclusive enough so that the circular test may be successfully applied. The base in this secondary sense of the basis of comparison should therefore be the whole group of places or interval of time included in the comparison. To avoid confusion it would doubtless be preferable to restrict the meaning of the term "base" to the former meaning, and to speak simply of the basis of comparison when the latter meaning is intended. For reasons already brought out, price indexes on a broader basis might not check with those on a narrower basis; but in each comparison expressed by a definite series of index numbers, the ratios should tally no matter which item may be chosen to represent 100 per cent.

Index numbers expressing changes in quantities or prices of a variety of commodities are usually derived by means of some kind of weighted average of the data. But if we are to derive such a formula by logical steps, it appears necessary to begin on some other basis. There are many different kinds of weightings, giving varying results, which appear to be applicable to the problem; but there is no way to discriminate logically among these weightings. While our final results may possibly be stated in the form of weighted averages, the approach to the required formula must be by some other route.

The task may best be approached by attempting to derive a formula for an index number of prices, since it will be found that a corresponding formula for quantities may conveniently be stated as a part of the same problem. In fact, the chief difficulty in deriving a composite index number of prices lies in the fact that the quantity units

are of various incommensurable sorts, and cannot be readily combined. The method of procedure may best be discovered by comparison with a simpler problem involving only one commodity. In such a case we would find relative prices in any two years merely by dividing the value sold each year by the number of units sold in the same year, and comparing the two results. The two quotients thus obtained would evidently be weighted averages of the prices prevailing during each year, respectively.

To illustrate, let us assume that data on the sales of wheat of a given grade for two successive days are at hand, and it is desired to find index numbers of the price of wheat for the two days in the market referred to. The procedure would evidently be as follows:

TABLE I

	Bushels (millions)	Price per bushel	Selling value (millions of dollars)	Average price (weighted)
First day	50 60 90 200	$1.10 1.15 1.20	55 69 108 232	$1.16
Second day	80 120 100 300	$1.25 1.20 1.28	100 144 128 372	1.24

As may be seen, the average price for each day is found by dividing the total selling value by the number of units sold. The results, $1.16 and $1.24, are weighted averages of the prices prevailing each day, and may be taken abstractly as index numbers, since they give correctly the ratio of the prevailing prices. Or, they may be reduced to terms of the first day as base, and written 100 and 107, or of the second day as base, and written 94 and 100, respectively. The whole procedure is simple and self-evident, and the results as expressed in terms of different bases are consistent.

But if with bushels of wheat we are compelled to combine tons of iron, yards of cloth, gallons of gasoline, and other dissimilar commodities, the procedure evidently cannot be so simple. It may, however, be analogous provided that we can find some way of combining the dissimilar units. If this can be done, then the aggregate selling price divided by the total number of units sold should give the average prices as before.

In order to combine the dissimilar physical units it is necessary to make use of the only available common denominator; namely, a unit

of value. If for each commodity the physical unit is revised to express the amount that ordinarily can be bought for one dollar, then the total number of these units for the various commodities may be aggregated as dollar's worths. Such a total of revised physical units, divided into the total value at actual selling prices, should give us a measure of the price level, just as the corresponding division for a single commodity gave us the average prevailing price.

The problem resolves itself, then, into a question of determining what shall be regarded as a typical dollar's worth of each commodity. At this point it is evident that we may get varying results according to the period of time which is made the base, in the sense of a basis of comparison, as previously discussed. From the standpoint of statistical theory, however, there is only one logical solution to this problem. The amount to be taken as a dollar's worth should be determined with reference to each of the units of time included in the comparison, making the whole interval the basis. Thus if we were comparing annual prices from 1914 to 1918, the typical prices would be the average prices for each commodity prevailing through the given years. To be more exact, they would be simple averages of the annual prices of each commodity, respectively. The justification for this procedure is found in the "theory of errors." In applying this theory each unit of goods sold during the year is taken as a determination of the price of the commodity in question for the year, and the annual prices in turn are taken as determinations of the typical price. In thus applying the theory the unit of time is necessarily considered to be concentrated at a single point, and slight modifications in the weightings may arise from changes in the length of this unit. Theoretically, a shorter unit of time is preferable to a longer, provided that the data are ample.

To be very precise, the computation of the typical price would require that the average price of each commodity for the series of time units should be found by the geometric rather than the arithmetical method. The reason for this requirement is that fluctuating prices tend to form a frequency curve that is positively skewed when plotted to an arithmetical scale, but that is normal when plotted to a logarithmic scale. A geometric mean would take this characteristic of the distribution into account. In practice, however, an arithmetic mean would be satisfactory, since the price index numbers finally obtained in this way would differ only in a negligible degree from those obtained by the use of the geometric average.

After the simple average of prices for each commodity through the whole interval of time has been found, the number of revised physical units of each commodity for each unit of time may readily be obtained

by multiplying the quantity by this price. The results will be dollar's worths, which may correctly be taken as revised physical units, since they represent the number of times that a dollar's worth would be contained in the quantity as originally expressed. The number of dollar's worths for each year may then be totaled to include all commodities, since they are alike with reference to a common standard of value, however dissimilar they may be in respect to the bushels, tons, yards, or gallons in which they are primarily expressed. The total of physical units expressed in dollar's worths may then be divided into the aggregate of values sold for each year, respectively, just as was done in the simpler case involving only one commodity. A series of such quotients will indicate the ratio of the prices prevailing in each unit of the whole time interval. The same method may obviously be applied to a series of localities as to a series of time units.

The method may be summed up in the following formula:[1]

$$P_n = \frac{\Sigma p_n q_n}{\Sigma p_m q_n}$$

in which,

P_n = a price index for any given year (or other time unit) in the series.
p_n = the price of any particular commodity for the given year.
p_m = the typical price of any particular commodity for the whole interval.
q_n = the quantity of any particular commodity handled in the given year, expressed in the physical units used as the basis of price.

When applied to each of the unit periods of an interval of time, the formula will give a series of price ratios usually averaging about one hundred per cent, and which may be reduced consistently to any desired base. The formula may also be written:

$$P_n = \frac{\Sigma \frac{p_n}{p_m} \times p_m q_n}{\Sigma p_m q_n}.$$

In this form the procedure is seen to be that of taking a weighted average of relative prices, the relatives being based upon typical prices, and the weights consisting of typical dollar's worths for the given unit of time. Thus the method may be reduced to a form of the weighted average, though it is a form that we would hardly have assumed on its face value to be more accurate than several other forms of weightings that might be proposed. The formula as first given is, however, more

[1] For a dual comparison this is equivalent to a formula first proposed by Walsh, and listed by Fisher as No. 2154. Fisher ranks it high for accuracy, and cites its factor antithesis as one of the most practicable and useful formulae.

convenient to use; and for purposes of illustration it is applied in the accompanying simple problem. It will be seen that the price indexes thus obtained vary above and below one hundred, having as their base approximately an average of the series. It will be seen also that a measure of quantities has incidentally been obtained, and that this measure may be taken as an index, capable of being expressed in terms of any desired base. The formula for a quantity index is simply the denominator of the formula for a price index; namely,

$$Q_n = \Sigma p_m q_n$$

in which Q_n stands for a quantity index for any unit of time in the whole interval, and the other notation is as before.

The formulae for price and quantity indexes as thus derived will meet the requirements of the principal tests that have been applied to such formulae. They are reversible or interchangeable as to the base taken as 100 per cent, since the formulae give the index numbers first simply in the form of ratio terms, which may readily be reduced so that any one of the terms becomes the base. That is, they meet the requirements of the time reversal and the circular tests. They will be found also to check up accurately for various arrangements of the original units. If, for example, a series of bushels is separated arbitrarily into two different series, no change will appear in the result. This test would not be met, however, if the typical price of each series of commodities were taken as a weighted rather than a simple average.

TABLE II

COMPUTATION OF VALUE, QUANTITY, AND PRICE INDEXES USING THREE IMPORTANT CROPS, UNITED STATES, 1914–1918

Year	1914	1915	1916	1917	1918	Average
Wheat						
a. Crop, U. S. (ten million bushels)	89	103	64	64	92	
b. Farm price per bushel (cents)	88	105	126	201	204	145
c. Selling value (ten million dollars)	78	108	81	129	188	
d. Dollar's worths at average price	129	149	93	93	133	
Corn						
a. Crop, U. S. (ten million bushels)	267	299	257	307	250	
b. Farm price per bushel (cents)	71	71	74	129	147	98
c. Selling value (ten million dollars)	190	212	190	396	368	
d. Dollar's worths at average price	262	293	252	301	245	
Cotton						
a. Crop, U. S. (hundred thousand bales)	161	112	115	113	120	
b. Farm price per bale (dollars)	43	46	72	109	141	82
c. Selling value (ten million dollars)	69	52	83	123	169	
d. Dollar's worths at average price	132	92	94	93	98	
Summary						
e. Total selling values (c)	337	372	354	648	725	
f. Total dollar's worths (d)	523	534	439	487	476	
g. Price level (e÷f)	64	70	81	133	152	100
h. Value index, base year 1914 (e)	100	110	105	192	215	
i. Quantity index, base year 1914 (f)	100	102	84	93	91	
j. Price index, base year 1914 (g)	100	109	127	208	238	

There is one test, however, among those proposed by Professor Fisher which is inapplicable to our results. This is the factor reversal test already alluded to. Professor Fisher assumes that the ideal formula for price index numbers should apply in exactly the same form to the computation of quantity index numbers, merely by interchanging the p's and q's. He assumes that this would be logical in that it would be a poor rule that would not work both ways. He argues that since no discrimination should be made among price items, no discrimination should therefore be made between price and quantity items. But this is a gratuitous assumption, and does not express a mathematical principle. Prices are not the same things as quantities, and we might expect on the analogy of mathematical formulae generally that each would call for a distinctive formula. It is true that the factor reversal test applied to our quantity index formula will give a practicable and fairly reliable price index formula, but it will not give a precisely accurate one. That price and quantity each requires a distinct type of formula is indicated by the simpler problem where only one commodity is involved, as in the case of the bushels of wheat previously discussed. In this case the measure of quantities for each period is obviously obtained merely by summing up the number of units sold, while the measure of prices is obtained by dividing the aggregate value by the quantity units. It would therefore be expected that the composite indexes derived by an analogous method would be associated with distinctive formulae for prices and quantities. Hence the factor reversal test may be disregarded.

There does not seem to be any escape, however, from the limitations imposed by the period chosen as the basis of comparison. Since typical prices for each commodity are a form of average for the whole period under consideration, the results will necessarily vary somewhat as the basic period varies. Prices in 1918 might be 200 per cent of those in 1914 when looked at merely from the standpoint of the war period, yet they might be measured by a slightly different index number on the basis of the whole interval of time since 1900. Such variations are inherent in the nature of the problem. Since quantities in composite indexes can be measured only by reference to a unit of value, they will shift somewhat as values shift, and as a result price indexes may be affected in some degree. Hence quantity and price indexes must be read with the understanding that they depend upon the viewpoint of a certain period taken as a basis of comparison. This basic period need not be the base in the sense that it is taken as 100 per cent, but it is fundamental to a given index series in that it affects the ratio of any two index numbers in the series to each other.

But although theoretically the basis of comparison should be the entire period covered by the given index series, yet practically speaking the rule may be modified according to circumstances without danger of material error. And for most purposes it is necessary to use a simpler formula which does not require a frequent revision of the quantities. As Professor Fisher has shown, there are several formulae which will give satisfactory results, although they are not logically precise and consistent. It is not the purpose of this article to add to or criticize these practicable formulae, but merely to consider the norm which they should approximate. The justification for the refinements of statistical theory in the search for index number formulae that are mathematically precise lies in the fact that the simpler formulae in actual use must be judged by reference to an ideal standard.

A NEW INDEX OF THE GENERAL PRICE LEVEL FROM 1875

By Carl Snyder
Federal Reserve Bank of New York

The concept of a definite and ascertainable general level of prices, or, what is the same thing, the average purchasing power of money, is relatively new. Though other centuries had acute experience of wide variations in this purchasing power, either through the debasement of the coinage or the discovery of gold, or other causes, it remained for the English economist, Jevons, to attempt the first extended measure of such changes.[1]

For this purpose Jevons took practically the only material then available, the quotations for prices of commodities at wholesale; and this has remained the practice of almost all of the standard books upon the subject, as those of Kemmerer, Fisher, Cassel, and the like. And in times of relatively slow change, as before the war, this method involved no very serious error.

But even for some time before the war there had begun a remarkable development of other types of price indexes, as of wages, cost of living, building construction, and the like, destined notably to sharpen our conceptions of price "levels." Then, after the war, came the wide "spread" between, for example, wholesale prices and wages, exactly as after our Civil War; so that our familiar indexes of wholesale prices, as the Department of Labor, Bradstreet's and Dun's, could no longer be considered an accurate measure of the General Price Level, or of the average "purchasing power of the dollar."

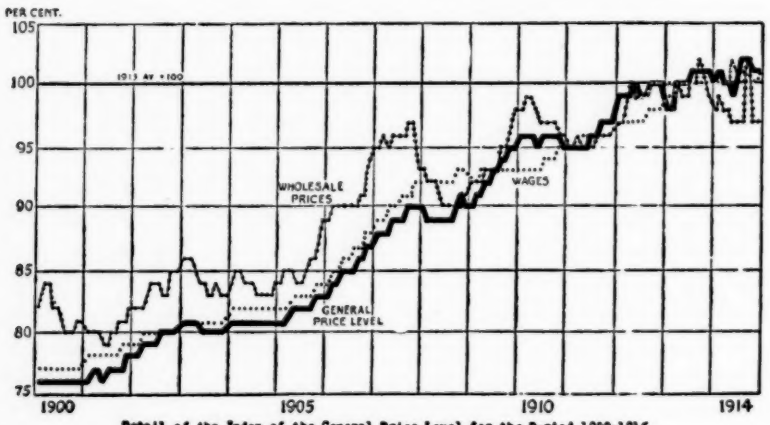
But how to weight up the various elements, to get at a true composite price level, was an extremely difficult problem. A solution of the difficulty, through the working out of a new index of the volume of the nation's trade, was indicated in the last number of this Journal.

In this country it is well known that a very large part, some 80 per cent or more, of payments of all kinds in the exchange of goods, property and services, are made by means of bank checks. It follows then that if we had some quantitative measure of all the things exchanged we could with it divide the total of bank clearings and thus compute the average of all payments; and this would be a measure of the General Price Level.

We do not know the exact total of all checks drawn, but we have so

[1] "A Serious Fall in the Value of Gold Ascertained" &c. 1863.

large a proportion of the total, some four fifths and more, that the balance is of slight consequence. For example, the aggregate of bank debits in about 240 cities, reported for the year of 1922, was 465 billion dollars; and from a comparison of the bank deposits in these cities with



Detail of the Index of the General Price Level for the Period 1900-1915, In Magnified Scale. (1913 = 100)

those for the rest of the country, we have estimated that the total amount of bank checks drawn that year, in the whole nation, was somewhere close to 540 billions; and for 1923 this aggregate would be about 6 per cent more.

We cannot utilize these debits directly, but we may do so in a roundabout way. It is obvious that if we could eliminate from bank clearings or debits the influence of changes in the price level, then in a broad sort of way the growth of clearings through a series of years would afford a measure of the growth of the nation's trade, since the larger part of the nation's trade is registered in these clearings. We should have to allow, also, for the increased use of checks as compared with hand currency; and there would also be the disturbing element of speculation in times of trade activity. As a matter of fact, throughout the last half century the increased use of checks has apparently been very slight. We had highly developed this device a long time ago. Then as to speculation, we find this largely centered in and represented in the bank clearings of New York City, which now form around 45 per cent of the estimated total of the country.

We have, from the work of Stewart, Day and Snyder, careful estimates of the growth in the total of the nation's production; in the last named case estimates for the entire period here covered. From all this we know that the industrial growth of the country has, for pretty

much the entire period under view, continued at the rate of very close to 3½ per cent per annum.

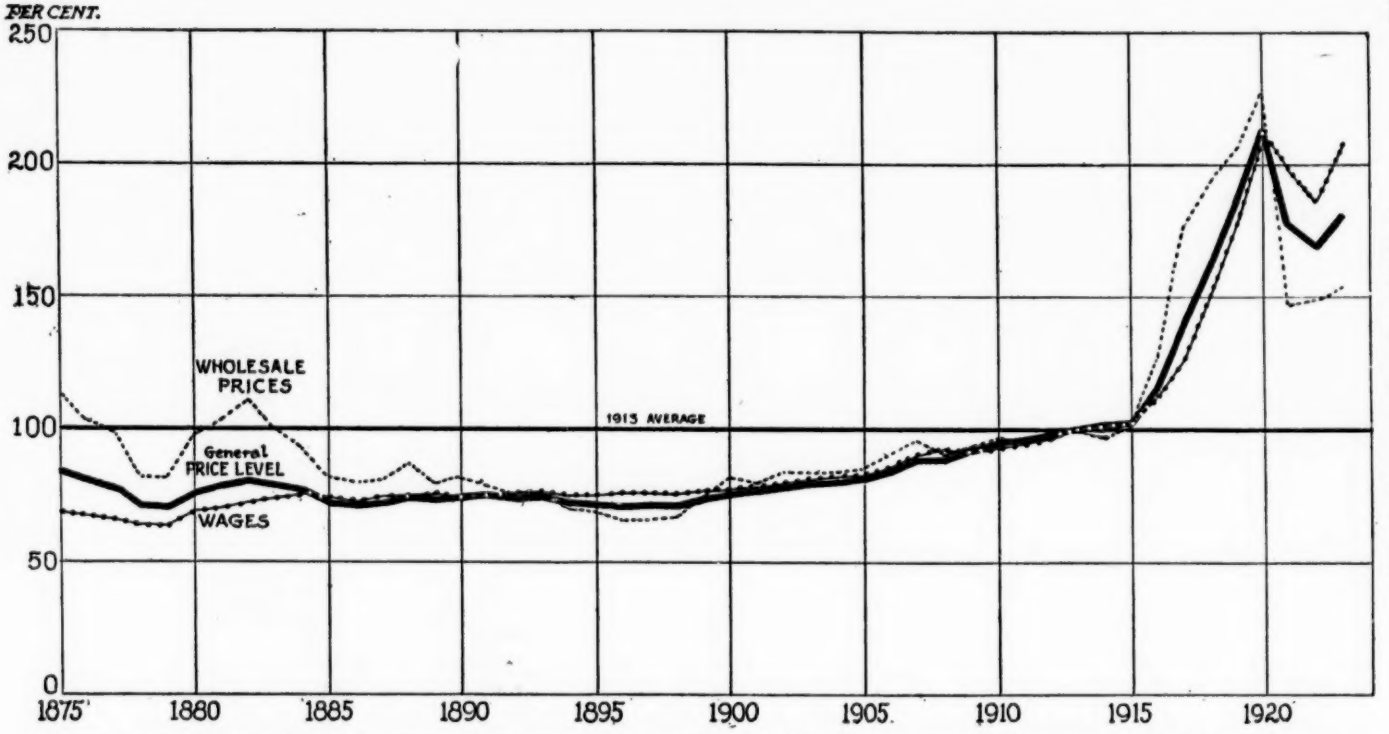
Taking the bank clearings reported from outside New York City as the best measure we have of the total volume of exchanges, we should expect to find, then, that the smoothed average of these over a sufficient period would increase at something like the same rate, if the element of price change could be eliminated. How, then, to measure this?

The most violent disturbance of prices in this period was, of course, in and after the late war. We know from the estimates of production that the increase in the war, over the normal rate of growth, was small and not greater than in what we call boom times. The test of a price corrector, then, would be that it reduce the inflated clearings of the last eight years so that the smoothed averages would continue the same trend as in the pre-war period.

Or we might proceed in the opposite way. Obviously the general price level represents some kind of a composite of the different indexes for commodity prices, wages, cost of living, etc. We do not know what proportions these different elements contribute to the total of bank clearings, but it is possible to make some sort of rough estimate of sufficient value for weighting purposes. On this basis an empirical index was worked out, giving equal weights to the indexes for commodity prices, wages, and for the cost of living, and one third as much to stock prices. This index, employed as a corrector for bank clearings, did continue the pre-war trend of the smoothed averages in a very remarkable way. But there remained another and more precise test.

In the last December issue of this JOURNAL I described a new measure of the total Volume of Trade[1] of the country, in quantitative terms or their equivalent. This new index of trade represented a weighted composite of 56 independently computed series presenting a wide sampling of the whole field of basic production, transport, wholesale and retail distribution, and various types of financial and commercial activity. Then, in the March issue, I described still another index of business activity which we have worked out from the variation in the activity or rate of turnover of bank deposits. The quite remarkable general concord between these two measures in the five years which are so far available seems to confirm the findings of the composite index of trade, so as to leave little doubt that, within the limitations therein set forth, we now have for these five years a reliable estimate of the actual variations of the nation's trade, in quantitative terms of goods and services and not of dollars.

[1] From the Reports Department of the Federal Reserve Bank of New York.



INDEX OF THE GENERAL PRICE LEVEL OR AVERAGE OF ALL PAYMENTS AS COMPARED WITH THE COURSE OF COMMODITY PRICES AT WHOLESALE AND A COMPOSITE INDEX OF WAGES (1875–1920), 1913 TAKEN AS EQUAL TO 100

Now if, as was assumed before the war, the variations in bank clearings represent a sensitive index of the actual variations in trade, the deviations of clearings corrected for price changes, from the smoothed average or trend, ought to correspond closely alike in percentage and in time to the variations in trade shown by these two new indexes of business activity. And in point of fact the line of bank clearings, corrected by the empirical index constructed as described, did correspond very closely. Obtaining this close agreement in this empirical fashion, it was then very easy, as Prof. Irving Fisher suggested, to shift the weightings a very little so as to make the agreement practically complete. The weightings finally employed are given in the appended note.

In a word, we have now three congruent series measuring changes in the volume of trade from month to month throughout the past five years. In the case of the third series, this congruence becomes the test of the make-up of an index of the general level of all payments by checks, which must be, in its turn, the closest possible approximation which we can now obtain to an index of the General Price Level. The chart herewith presents such an index worked out from these materials for the last half century. For the purpose of comparison with this are given the index of commodity prices at wholesale, and also a composite index of wage payments.

Of a number of points of interest the following may be noted: First, as might reasonably be expected, in and after the war the general level did not rise anything like as high as, for example, commodity prices at wholesale; and in the great decline which followed did not fall anything like as much; showing, in other words, far greater stability. Just as many individual commodities in this same period rose and fell in price in far greater degree than did the general average of commodity prices at wholesale, so the averages of wholesale prices swung much above and then much below the larger composite of all prices.

For the last year and a half or more there has been a wide and continuous spread between these two; and from this we note that the depreciation in the average purchasing power of the dollar from the pre-war level has been not around one third, as is currently assumed from the position of the Department of Labor index of commodities, but much more than this; to be accurate, very close to 45 per cent.

It may be added that the index of the general level runs fairly well, as a rule, with the current indexes of the average Cost of Living in a worker's family; and as the composite index of wage payments has for some time ranged some 15 or 20 per cent higher than this, we have here a pretty clear explanation of the increased purchasing power of wage earners for new houses, automobiles, radio, etc., in the last few years.

The other fact which stands out in very bold relief is the relative stability of prices throughout the first forty years of this period as compared with the succeeding ten. It is interesting now to reflect upon the great agitations of this earlier period, and the basis upon which they were founded. The continued fall in prices which followed the depression of 1873 (and the "Crime" of the same date) gave rise to an enormous literature and to profound political perturbations. Similarly, as early as 1903 the advance of prices which followed had occasioned commissions of inquiry as to the causes of the increased cost of living; and by 1910 or 1912 this rise had volleyed into a national outcry.

Yet, if we take the average for the forty pre-war years, we shall see that the decline below this average never amounted to more than 13 per cent, for the annual averages; nor had the rise by 1913 amounted to more than about 25 per cent. Changes of this moderate degree, through a long series of years, evoked a far greater protest than has the great dilution of currency value brought about in and since the war. The changes of the pre-war period were regarded as an act of injustice and a moral wrong; those of the war period as a fatality. It can scarcely be said that, in this half century, monetary theory has made any serious inroads upon the national Mind.

From the study of the variation in the rates of deposit turnover already referred to, it would appear that, as velocity seems to vary closely with the volume of general trade, we have no need to assume also a cyclical variation in price, with business, as many have believed. Some types of prices, as of certain commodities (as Professor Persons has shown) may have such a rhythmic variation; but in the preliminary study of the General Price Level given in this article no such cyclical variation in the average of all prices seems evident. If any such exists, it would require more precise methods of measurement to disclose it.

COMPOSITION OF THE INDEX

The different factors comprising the new index of the General Price Level have been weighted as follows: Commodity Prices at Wholesale, 2; Composite of Wage Payments, 3½; Elements of the Cost of Living, 3½; Rents, 1. An effort to include an index of security prices was not successful. Sources of material as follows:

Commodity Prices at Wholesale: 1875–1889, Monthly Index of Basic Commodities compiled by Federal Reserve Bank of New York; 1890–1899, monthly figures interpolated from quarterly series of the United States Department of Labor; 1900 to date, Department of Labor monthly index.

Wages: 1875–1913, Department of Labor annual index of unskilled labor; 1914 to date, earnings in New York State factories and Federal Reserve Bank index of Average Weekly Hiring Rate for Unskilled Labor. Wages of teachers and clerks, 1875–1913, Burgess' "Trend of School Costs"; 1914 to date, average wages for teachers in the United States, Research Bulletin, Journal of the National Editorial Association,

March, 1923. Wages of clerical workers in New York State factories, 1914 to date, New York State Department of Labor.

Cost of Living: 1875–1889, estimates of family budgets compiled by Russell Sage Foundation; 1890–1909, Cost of Living index estimated from retail food index of United States Department of Labor; 1910 to date, United States Department of Labor index for 32 cities; monthly figures interpolated from data published by Massachusetts Commission on the Necessaries of Life.

Rents: 1875–1913, special study on rents by Russell Sage Foundation, unpublished; 1913 to date, Shelter Index compiled by United States Department of Labor; monthly figures interpolated between annual and quarterly figures.

Credit for the compilation of this index is largely due to Miss Irene Sheehan, Reports Department, Federal Reserve Bank of New York.

INDEX OF GENERAL PRICE LEVEL BY MONTHS

Years	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Average
1875	84	84	84	84	84	84	84	84	84	83	82	82	84
1876	82	82	81	81	80	79	80	80	79	79	79	79	80
1877	79	79	79	78	79	78	78	77	76	76	75	74	77
1878	74	74	73	72	71	70	71	71	70	70	70	69	71
1879	69	70	70	69	69	70	70	70	70	72	73	74	71
1880	77	77	77	76	76	75	75	75	76	76	77	77	76
1881	77	77	77	77	78	78	78	79	79	81	81	81	79
1882	80	80	81	82	82	82	82	83	81	80	80	79	81
1883	79	80	80	80	79	79	78	78	78	77	77	77	79
1884	78	78	77	77	76	76	76	76	76	75	74	74	76
1885	73	73	73	73	74	73	74	74	74	74	73	73	73
1886	73	72	72	72	72	72	72	72	72	72	72	73	72
1887	72	73	73	73	73	73	73	73	73	73	74	75	73
1888	75	75	75	75	75	75	75	75	76	76	76	75	75
1889	75	74	74	74	74	74	74	74	74	74	74	74	74
1890	75	75	75	75	75	75	75	75	76	76	76	75	75
1891	76	76	76	76	76	76	75	75	75	75	75	75	76
1892	75	75	75	75	75	74	74	74	74	74	74	74	74
1893	75	75	75	75	75	75	75	75	75	75	75	74	75
1894	73	73	73	72	72	72	72	72	72	72	72	72	72
1895	72	72	72	72	72	72	72	72	72	72	72	71	72
1896	71	71	71	71	71	71	71	71	71	71	71	71	71
1897	71	71	71	71	71	70	70	71	71	71	71	71	71
1898	71	71	71	71	71	71	71	71	71	71	72	72	71
1899	73	73	74	74	74	74	74	74	75	75	75	75	74
1900	76	76	76	76	76	76	76	76	76	76	76	76	76
1901	76	76	77	77	76	77	77	77	77	77	78	78	77
1902	78	78	79	79	79	79	80	80	80	80	80	80	79
1903	81	81	81	81	80	80	80	80	80	80	80	80	80
1904	81	81	81	81	81	81	81	81	81	81	81	81	81
1905	81	81	81	82	82	82	82	82	82	82	83	83	82
1906	83	84	84	85	85	85	85	85	86	86	87	87	85
1907	88	88	88	88	89	89	89	89	90	90	90	90	89
1908	90	89	89	89	89	89	89	89	89	90	91	90	89
1909	90	90	91	91	92	92	93	93	94	94	95	95	93
1910	96	96	96	96	95	95	96	96	96	96	96	95	96
1911	95	95	95	95	95	95	96	96	97	97	97	97	96
1912	99	99	99	99	100	99	99	99	100	100	100	100	99
1913	98	98	100	100	100	100	101	101	101	101	101	101	100
1914	100	101	100	100	99	100	101	102	102	101	101	101	101
1915	102	102	101	101	102	102	102	103	103	105	106	106	103
1916	109	109	111	112	113	114	115	116	119	121	124	126	116
1917	128	129	131	135	139	141	141	142	145	147	148	149	140
1918	151	153	155	157	159	163	166	169	174	176	175	175	164
1919	179	176	177	178	180	181	185	189	189	193	198	201	186
1920	207	207	210	215	219	219	221	218	218	213	208	201	213
1921	196	190	186	182	179	172	173	174	173	172	171	171	178
1922	167	166	167	167	168	169	170	171	172	173	175	176	170
1923	177	178	179	180	182	182	183	182	183	184	183	183	181
1924	181	181	181										

THE NATURE OF AN UNEQUIVOCAL PRICE-INDEX AND QUANTITY-INDEX

PART II

BY SIR GEORGE H. KNIBBS

20. The Principle of the Vicarious Equivalence of Regimens. When the fluctuations in price or in quantity of any given series of commodities A, B, C, etc., are relatively identical with those of some other series A, B, C, etc., the former may be said to be the price-fluctuation equivalent or the quantity-fluctuation equivalent—as the case may be—of the latter. When the fluctuations of the product pq are relatively the same they may be said to be the economic-fluctuation equivalent. And finally when the values pq for A, B, C, etc., are identical with those for A, B, C, etc., the former may be said to be the complete economic equivalent of the latter. Their substitution would leave the value of deduced indexes unaffected. When, therefore, we are compelled, owing to the flux of things, to change the regimen, the introduction of complete economic equivalents does not prejudice the results practically. This may be called the principle of vicarious equivalence.

These equivalences in commodities, viz.: of (i) price-fluctuations; (ii) quantity fluctuation; and (iii) complete economic fluctuation are, of course, in actual cases, never completely fulfilled, but they are partially so quite frequently, and in proportion as they are, the limitations arising from differences in the basis of comparison tend to disappear, so far at least as any practical uses of price-indexes are concerned. We may envisage the situation in the following way:

While it is true that, when a complete change occurs, between any two dates, in the elements of a regimen, used as a basis for ascertaining price-indexes, such change necessarily renders the significance of the deduced index somewhat dubious, there are nevertheless at least two cases in which the deduced index may—within proper limits—be regarded as quite valid, viz.:

(a) When there is a vicarious equivalence in respect of substituted commodities (such a substitution may be progressive, alternating, or sudden), and

(b) When the variations in the exchange-value are identical for the different regimens.

In regard to (a) it may be said that commodities may be vicariously equivalent in several ways; for instance:

(i) Through industrial and chemical advances leading to improvements in quality, finish, grade, purity, etc., so that the original commodity is slowly displaced by a higher grade (*e. g.*, refined sugar displacing unrefined).

(ii) Through one class of commodities more or less rapidly replacing another (*e. g.*, through change of custom in regard to food, to clothing, to habitation, etc.).

(iii) Through trade, industrial, productive, or commercial conditions compelling a change in the various commodities used (*e. g.*, substitution of margarine for butter, of the flesh of fish for that of oxen, sheep, etc.).

In regard to (b), viz., to the second type of variation, it may be said that ordinarily it cannot be sharply defined, but may be described in the following way. Let us suppose that the prevailing usages of commodities, in two countries or communities, though not identical, are constant in each, and that their price-indexes exhibit identical fluctuations. It would then follow—as already said—that the general relationship between money and commodities was identical in each, despite the fact that the usage of commodities (*i. e.*, the regimens) was dissimilar. Or to state the matter differently, the exchange-relation between money and commodities may be identical, although the bases revealing this identity may be different.

It is this last consideration which often raises the question "whether it would not be better to employ all means which exist for measuring the fluctuations in the relation between money and commodities," say by the price-ratio method, which however has already been considered and rejected as unsatisfactory. It will suffice to observe here, that because some approximation in different regimens to vicarious equivalence often exists and because also properly computed indexes are dependent only upon relative changes of the elements of a regimen *inter se*, that considerable changes in a given regimen may not greatly change an index. Moreover so long as the ratios among themselves of the absolute amounts of usage are sensibly unchanged, the computed price-index is unaffected. Similarly, if the prices of a regimen on two different dates, remain among themselves in the same ratio, the quantity-index is unaffected, provided, of course, that the mean of the prices of each element on the dates compared is used throughout.

21. The Applicability of Price-Indexes. In every economic application of a price-index, the question arises "does the regimen constituting its basis substantially represent the relevant facts?" One phase of this

matter is considered later in regard to the application to questions of the cost of living. As already indicated price-indexes do not exactly apply to individual cases, for each individual has his characteristic regimen. Not only so but each group also has its characteristic regimen whether the group be a family, a community, or a nation. Moreover, the actual regimens differ with the circumstances of a community, for example, according to its health, employment, economic situation, the season, the locality, etc. It is, of course, both impracticable and unnecessary to take everything into account; one is forced, and it is sufficient, to adopt a regimen, representing substantially the mean for an adequate period of time, covering all minor fluctuations—in short a regimen for general usage, representing the average habit of a considerable number of individuals. Indexes thus have reference, not to the individual, but to the average man under the average conditions. To attempt more than this is to defeat the general purpose.

But if, in the course of time, the actual average regimen should differ sensibly from the regimen adopted to deduce a price-index, the deduced index loses its basis in reality; in short, it applies to a supposititious and not to an actual case. It is for this reason that in order to be sufficiently representative of the actual facts, a regimen has perforce to be adjusted from time to time, losing however none of its generality.

In this connection, however, it is necessary to remember that absolute quantities do not affect the result, provided the same quantities are employed in finding the aggregates of the basic date as well as for the second date. Thus, as indicated in Section 4, the quantities could be doubled, trebled, etc., without altering the price-index, in other words we might use for, the quantities of the regimen mq_a, mq_b, mq_c, etc., instead of q_a, q_b, q_c, etc. But we cannot use kq_a, lq_b, mq_c, etc., without vitiating the ascertained index, unless k, l, m, are so nearly equal as not sensibly to affect the result. It is this that leads to considerations of the means by which indexes may be made virtually to adapt themselves to the changing circumstances of a community.

22. Application of Price-Indexes to Questions of Cost of Living. Estimates of the cost of living according to some adopted standard made at any one date are deducible for any other date, provided the price-indexes for the two dates are known, and the adopted standard remains a reasonable one. Thus if C_0 is the cost of living at the basic date and C_1 at some other date, then on the assumption that cost has not affected the regimen, we have

$$(18) \qquad \ldots \quad C_1 = C_0 I_1 / I_0, \text{ or } C_1 / I_1 = C_0 / I_0,$$

which must also be sensibly true for any other regimen similar to the adopted one. The validity of the application rests upon the idea that the commodity-purchasing power of C_1 equals that of C_0, and this is represented by the second form of the above equation.

The ascertainment of cost of living at the original date, however, is not a simple matter because it is inherently a very elastic quantity, *i. e.*, it is one which depends upon purely arbitrary and ever-changing standards, and upon personal characteristics in respect of thrift and intelligence. Consequently estimates to be of any value, or entitled to be considered well ascertained, are very difficult of determination. This is due in particular to the following circumstances, viz.:

(i) The irregular nature of fluctuations in the prices of commodities.

(ii) The fact that such fluctuations tend to modify, temporarily or permanently, the usage of commodities.

(iii) The usage of commodities is variable in regard to their absolute amounts, and as regards the amounts *inter se*.

(iv) The usage is modifiable between somewhat wide limits owing (a) to the variations in thrift, and (b) in the degree and propriety of substitutions of one commodity for another under the compulsion of cost or of temporary lack of particular commodities.

In regard to (i) it may be noted that the prices of commodities are subject to (a) regular (periodic or seasonal), and (b) irregular variations, the latter often tending to, or even actually quite masking the former. Thus the standard regimen by which the price-index is measured is based rather upon an average condition than upon an instantaneous one.

In regard to (ii) it may be observed that fluctuations of price react upon the amounts used, and may sometimes lead to permanent changes therein.

Respecting (iii) it may be said that—other things being equal—price fluctuations when considerable tend to alter the quantity ordinarily used of particular commodities, to cause substitutions of one commodity for another, to lead to the appearance of new commodities and even to the disappearance of others. For example, margarine may have at times to be substituted for butter wholly or partially.

In regard to (iv) it should be noted that expenditures upon living, *i. e.*, upon food, clothing, housing, entertainment, etc., are modifiable within very wide limits indeed without detriment to the user; a fact revealed by the very different way in which different individuals within a community can live upon given amounts.

Lastly it may be observed that variations in seasonal temperatures,

etc., and in employment, greatly alter the food necessities of the organisms; hence, even for the same individual, there may be both periodic (seasonal) and irregular changes, upon which the cost of living might very properly be estimated.

For these reasons the price-index is to be regarded as founded on a wide basis, so as to serve appropriately as an indicator of the factor of the general purchasing-efficiency of money. Thus it is to be taken into account, only as one of the many economic factors which call for review in estimations of the changing cost-of-living. The notion that a cost-of-living inquiry necessitates a recognition of only the change in the general purchasing-efficiency of money, loses sight of the fact, of course, that the two questions are distinct.

Standards-of-living are quantities which actually merely hover about fixed values, but are themselves not actually fixed. Variations in the scarcity of a commodity involve variations in the regimen. Moreover, commodities vary in their relative importance; essential foods are of different degrees of intrinsic interest in a regimen, and in adopting them, other than economic measures may have to be regarded.

The elements of cost-of-living are essential foods, clothing, and housing, each of which have a general variation with time, and also one of kind. Even the element of firing varies, as also does the caloric element in food and the heat value of clothing. Obviously the variations of regimens can be made almost as meticulous as one pleases or can be treated as an average the period of which is one year.

23. Different Bases for the Measurement of Purchasing-Power. For every characteristic type of community, for every range within such type, and for any type at different times, a regimen exists which would serve as a true expression of its usage. In brief, the regimen is a function of communal character and time. In making comparison indexes such continuity as is possible can be secured by making the regimens change and adopting one regimen for two successive dates, applying the prices at those dates and comparing the aggregate costs. Only the closest possible regimen common to the two compared dates can furnish the appropriate index. Comparisons between indexes computed for widely separated dates are subject to great limitations, and are invalid if a common regimen to serve as a basis cannot be assumed to exist. The notion that other schemes of computation are valid is based upon fallacious reasonings. The whole foundation of an unequivocal price-index is that the index I_{01} is $\Sigma p_1 q_m / \Sigma p_0 q_m$, q_m denoting the regimen which may appropriately be assumed as common to two compared dates; no other index really unequivocally measures

the reciprocal of the ratio of the purchasing-power of money. Moreover, this purchasing power depends upon the group of things and the relative quantities thereof purchased, hence the range of its validity is limited by the applicability or otherwise of the adopted regimen.

The question may, however, fairly be asked whether price-indexes based on aggregate expenditures, are not uncertain since fluctuations in price themselves tend to cause an actual variation in the regimen adopted for the purpose of computing them. We have already seen that the price-ratio method is really valid only when in agreement with the aggregate-expenditure method; that is, when the relative weights of the different price-ratios are those, based on the ratio of the aggregate-expenditures, for the quantities constituting the regimen at the date taken as the basic date.

24. The Equivocal Character of all Other Price-Indexes. Some definite index, J say, which can be deduced—let us suppose—in a variety of ways, would of course have a definite meaning for each one of them. Thus if at any given date the expenditures, according to n different regimens, were identical, then we should have n different bases upon which to judge the variations in the purchasing power of money yielding a like total at this date. And—to suppose what practically never occurs—if at some other date, the expenditure according to the same n regimens all gave an identical result (either in agreement with, or differing from the former), we should have learned that, according to n different regimens, there were n identical values of the index J, each unequivocal. In other words, we should have ascertained that according to n differing schemes of expenditure, the results were the same. This, as said, does not occur in practical cases, but often the differences are less than inexperienced persons would have imagined. This arises largely from the fact that the absolute amounts in a regimen do not matter; they affect the numerator and denominator in the same ratio. Secondly very considerable variations among the elements, *inter se*, often produce relatively small differences in the ratio of each pair of aggregates; that is, if P and Q, and p and q are two different regimens (prices and quantities) $\Sigma P_1Q_m/\Sigma P_0Q_m$, are often very nearly $\Sigma p_1q_m/\Sigma p_0q_m$. But this does not affect the principle that a price-index must not be affected by variations in the regimen; to be unequivocal it must reflect only the effect of variations of price, not variations both of price and quantity. It is not properly called a price-index if it does. If this were kept clearly in view much of the invalid reasoning which has misled economic writers would have been avoided.

25. Confused-Indexes. Indexes which reflect changes in the regimen

as well as changes in price may very appropriately be called confused indexes. Their intrinsic nature may easily be illustrated. Suppose the quantities used in a unit of time of commodities A and B were at one date a_0 at price α_0 and b_0 at price β_0, and at a second date a_1, α_1 and b_1 and β_1, so that the aggregates were as in the denominator and the numerator of the expression hereunder:

$J_{01}=(a_1\alpha_1+b_1\beta_1)/(a_0\alpha_0+b_0\beta_0)=T_1/T_0$, then J_{01}, is a confused-index; it reflects changes both in price and in quantity.[1] Now a confused-index has no definite meaning; nor is the confusion avoided by using price-ratios. The price-ratios are α_1/α_0 and β_1/β_0. If their weights are based upon the expenditures relative to the total expenditure on date 0, then, writing T_0 for the aggregate, and A_0 and B_0 for the component expenditures, at date 0, they are A_0/T_0 and B_0/T_0; if based upon the expenditures on date 1, they are A_1/T_1 and B_1/T_1. These obviously give different results and the results have no definite meaning. We may illustrate by an example:

Taking the example in the preceding footnote, the weights would be 18/38 and 20/38, and 80/179 and 99/179 respectively. The price-ratios are 5/6 and 9/8. Using the first, the computed price-indexes are:

$$\frac{18}{38}\times\frac{5}{6}=\frac{15}{38};\quad+\frac{20}{38}+\frac{9}{8}=\frac{22\frac{1}{2}}{38};\quad=\frac{37\frac{1}{2}}{38}=0.9868,$$

and using the second, the indexes are:

$$\frac{80}{179}\times\frac{5}{6}=\frac{66\frac{2}{3}}{179};\quad+\frac{99}{179}\times\frac{9}{8}=\frac{111\frac{3}{8}}{179};\quad=\frac{178\frac{1}{24}}{179}=0.9946.$$

Both these (supposed) price-indexes are less than unity, though not identical; the relative costs of the regimens, however, are 1.1776. If the regimens had been the same on the two occasions and the mean of what they actually were, the ratio of the expenditures would have been 0.9792. This last is the price-index and is the only index of an unequivocal character. It means that, had the quantity-usage of the commodities been the mean of that on the two dates, the cost on the second date would have been only 0.9792 of that on the first date. Thus a price-index has been deduced. The others are not price-indexes at all, but indexes in which the effects of change of price and change of quantities used, are confused.

[1] Thus suppose at one date 6 lbs. of bread at 3*d*. per lb. and 5 lbs. of sugar at 4*d*. per lb. were used and at a later date 8 lbs. of bread at 2½*d*. and 5½ lbs. of sugar at 4½*d*. per lb. were used. J_{01} would be (20+24½)÷(18+20)=1.1776: this would be a confused index. The best price-index would be based on 7 lbs. of bread at 3*d*. then at 2½*d*., and 5½ lbs. of sugar at 4*d*. and 4½*d*., that is, 42 and 41½. The latter divided by the former is 0.9792. This is an unequivocal index and is based on the average regimen.

It is important to notice that the true index is not the mean of the two values of those computed from price-ratios using their weights initially, or finally. If however we take, say, the mean quantities and base the weights on the ratios to the total expenditures at the first date, then we shall get the same price-index, viz., 0.9792. (See Section 18.)

It is now easy to see that merely because we follow some elaborate technique, we do not get rid of the effect of fundamental misconceptions; nor do we improve the significance of results because we adopt a technique which hides the real unsatisfactoriness of the character of the method followed. The preceding example is of course not intrinsically altered by introducing a larger number of commodities than two.

It is also easy to see that the reasonings lying behind deductions of indexes from price-ratios are usually erroneous. It would, for example, be erroneous to argue as follows: The variation of price for commodity A in the ratio ρ for commodity B, in the ratio σ, and so on; therefore, since they are of the weight or importance r, s, etc., we are justified in adopting $(r\rho+s\sigma$ etc.$)/(r+s+$ etc.) as the general variation of price. Such reasoning would be unsound unless the weights are ratios based upon a fixed series of quantities, with the prices as at the basic date. If they are not this, the resulting value obtained for the price-index is "confused," that is, it reflects both changes in price and quantity.[1] To be satisfactory it of course should not do so. The conditions which govern the rigorous use of price-ratios were indicated in my article in *The Australian Labour and Industrial Report* No. 1, 1912, pp. xlii, xlvi to xlviii, and are already referred to in Section 18.

26. Independent Agreements of Price-Indexes. Suppose that we had two series of commodities, A, B, C, etc., and R, S, T, etc., and that the price-ratios obtained from each were ρ, ρ', ρ'', etc., and σ, σ', σ'', etc., and that these properly weighted, and the sum of the weights being unity, gave for the price-indexes

$$(19) \qquad \ldots \quad I=\rho_0=u\rho+u'\rho'+u''\rho''+\text{ etc.},$$

$$(19a) \qquad \ldots \quad J=\sigma_0=w\sigma+w'\sigma'+w''\sigma''+\text{ etc.},$$

then if $I=J$ it might be said that there were two cases where three changes of the purchasing-power of money was the same. These

[1] Prof. F. Y. Edgeworth's article on Mr. Correa M. Walsh's "The Problem of Estimation" does not correctly state the intrinsic nature of the problem, when he says we "deal with samples only of the things we buy, and the practice of sampling is governed by the theory of errors," see *Journ. Roy. Stat. Soc.* Vol. lxxxvi, 1923, p. 570.

could apply to two series of commodities in one community, or to a series in each of two communities. This, however, merely shows that two quite different regimens may perchance have the same variations in cost. It does not, however, show that the usage-regimen is unimportant, and it may be added that it is certain that any parallelism of variation will only be temporary. Nor does it follow that in large comparisons price-ratio methods have some advantage. If we deduce a regimen which is applicable over a suitable length of time, and apply existing prices to its elements, then the ratios of the aggregates $\Sigma p_n q$ give a result that in all cases is as good as any which can be had and one which has the advantage of being intelligible in every respect.

27. Comment on So-Called Ideal-Indexes. Professor Irving Fisher supposes that the geometric mean of two indexes, one computed from the initial regimen, and the other from the final regimen, has some special merit. He proposes therefore

$$(20) \qquad . \ . \ . \qquad I''_{01} = \sqrt{(\Sigma p_1 q_0 / \Sigma p_0 q_0)} \cdot \sqrt{(\Sigma p_1 q_1 / \Sigma p_0 q_1)}.$$

For one commodity this simply gives p_1/p_0 and is quite independent of q_0 and q_1, which however it may be said is assumed to be $\sqrt{(q_0 q_1)}$, on each occasion. For two commodities it may easily be shown to be equivalent to

$$\sqrt{\left\{\frac{p_1 q_0 + p'_1 q'_0}{p_0 q_1 + p'_0 q'_1} \cdot \frac{p_1 q_1 + p'_1 q'_1}{p_0 q_0 + p'_0 q'_0}\right\}}$$

each factor being obviously unsatisfactory though in opposite ways. On the other hand if we change the denominators it is still unsatisfactory. The result is not unequivocal because one does not know the exact regimen to which it corresponds. It is far better to make out a definite regimen, say q_m, based on q_0 and q_1, and use this with prices p_0 and p_1. Thus taking the example in footnote to Section 25, we get according to (20)

$$I''_{01} = \sqrt{\left(\frac{75}{76} \cdot \frac{179}{184}\right)} = 0.9600,$$

while had we taken the mean usage as the controlling regimen, we should have had

$$I_{01} = 41\tfrac{1}{8} \div 42 = 0.9792,$$

which is quite unequivocal. See footnote to Section 25.

28. Conclusion. The general result is as follows:

(i) The only unequivocal index is that computed by applying the prices of the compared dates to a definite regimen, and ascertaining the ratio of the aggregate values.

(ii) The only question in any case which needs consideration is what regimen should be adopted as the basis on which the computation may be made.

(iii) The best regimen is that in which the quantities are the means of their values at the compared dates.

(iv) There is no possibility of a real continuity of indexes, except on the basis of a common regimen which sufficiently accurately represents actual usage over the whole of the period to be compared by a system of indexes.

(v) All indexes extending beyond such a period, and therefore founded on a different regimen, are strictly not directly comparable.

(vi) Economic studies involving the use of price-indexes which are not unequivocal, must be founded not alone on indexes but other appropriate considerations must be taken also into account.

NOTES

TEN YEARS' EXPERIENCE IN BUSINESS STATISTICS

By H. Parker Willis

In the history of statistical effort, a large space ought to be allotted to the work of those who have sought to compile data that are of no value. Another large space should be given to the expedients that have been employed for the purpose of filling "gaps" where no statistics existed, and where none could be obtained. During the past few years in which the study of business statistics has become popular, the lack of accurate foundations for deductions and the still greater inadequacy of available data for use in making "forecasts" have long been recognized as elements tending to vitiate most of the work that was being done. Particularly has this been true in banking and credit, a field in which frequent, accurate and well-chosen information is indispensable. There may be some use for conjectural estimates of population by five-year intervals designed to fill a twenty-year gap during which no enumeration took place, but that method certainly will not answer the purpose in any statistical examination which seeks to provide a basis for business policies.

STATISTICAL CREDIT STUDY

During the decade from 1912 to 1922, it fell to my share, in connection with the drafting of the Federal Reserve Act and later the preparation of organization plans for installation of the Federal Reserve banks, to direct the preparation of much tabular material bearing upon banking and business conditions. More than ever before, I realized the necessity of reliable and extensive statistical information bearing upon prices, banking and credit, and I came more than ever to recognize the fact that without such information, credit analysis must be largely inadequate, confined entirely to the experiences of sundry individual businesses, and without the possibility of references to credit types representing the average or normal conditions prevailing in those particular businesses at any given time. The necessity of this kind of statistical apparatus as an auxiliary to sound bank credit analysis has lately been recognized in an increasing degree, but even as yet is ignored by many bank credit men of the older school. It has, if anything, been even more difficult to bring about an admission of the need

for more frequent bank condition figures and a sane and scientific use of those that are available, than to get the business statistics that were needed, yet large progress has been made in that direction as well. Probably the hardest work in the advancement of business statistics has had to be done in encouraging the compilation and classification of real fact data concerning production, transportation and distribution in the commodities field. Even there, however, some advancement has been attained, and the science of real business analysis has made marked progress. The recent conditions when, due to the efforts of an attorney-general not greatly distinguished for wisdom, a kind of prohibition has been imposed upon even the preliminary gathering of business statistics of the sort required are reminiscent of the period in Old Testament history when orders were given under no circumstances to take a census of the population.

RECALCITRANT STATISTICIANS

And yet this period of advancement, important as it has been, has had its own problems and difficulties and among them some which were produced or at least aggravated by the recalcitrant or non-coöperative attitude of statistical authorities themselves. We have today in the United States two very well-marked schools of thought. Division between them may be unconscious,—the line of demarcation may not be well-recognized,—but it is there. One group believes that the more fruitful and valuable work to be done is to be accomplished by the manipulation of data already available. Doubtful mathematical generalizations, often made by "experts" with but limited mathematical knowledge, the drawing of "curves," the making of composite "index numbers," and other efforts of the same sort constitute the staple of the effort of this group among our statisticians. The other group takes the view that what is needed above all else is fact ascertainment and fact analysis; that the gaps in our knowledge should be filled as thoroughly and accurately as may be, that the information thus obtained should be used in conjunction with existing information, and that, above all else, generalizations and forecasts should be withheld until the fact basis upon which alone they can be safely founded has been ascertained. So far as I have been able during the past decade to contribute to the work of the latter school, I have considered it a duty to do so, and at the same time to oppose hasty generalization and confident forecast based on inadequate data. This article is intended as a brief sketch of some of the efforts and some of the failures that have occurred during the ten years in question, and as an analysis of difficulties and problems thrown into light by this experience.

BETTER BANKING FIGURES

I have always thought that the banking statistics of the world were regrettably inadequate, and that particularly in a country like the United States, where "free banking" is the rule, the figures furnished by the Comptroller of the Currency were quite too limited for the purposes they were designed to serve. My first keen realization of this broadly-apprehended fact came in connection with the effort to prepare, in connection with the first drafts of the Federal Reserve Act, careful figures showing the reserves and general banking power of the several parts of the country, and to accumulate figures which would throw light upon the various banking questions produced by the division of the country into banking districts. The creation of a central banking system necessarily implied the introduction of a more careful set of statistics and of statements concerning its condition. Hence the introduction into the original Federal Reserve Act of a mandatory provision calling for the publication each week of a satisfactory statement concerning Federal Reserve condition, and the annual publication through transmission to Congress, of a report of the Federal Reserve Board designed to give a complete account of the operations of the system. When the work of organizing the Federal Reserve System began, and the preparation of the plans therefor was entrusted to a special committee of which I became chairman, very considerable time was devoted to the development of projects, subsequently in the main realized, for obtaining a weekly report of banking conditions which would possess genuine value. Closely allied with this endeavor, was the effort (in a much lower degree successful) to obtain a regular and systematic report of banking operation and of credits extended, for daily transmission to the Federal Reserve Board, there to serve as a foundation for a scientific study of credit throughout the country.

After the organization of the Federal Reserve Board, the problem of communication between that Board and the member banks became urgent. The Act had necessarily left the decision in this matter very largely to the Federal Reserve Board itself, and that body had come to the conclusion that no hasty action should be taken. Yet as time went on, it became more and more evident that the Board must receive frequent and systematic reports of condition, if it were to be expected to maintain any real supervision over the several banks. Hence the eventual decision to obtain from Federal Reserve agents as chairmen of their several banks, not only an annual but also a monthly report, and finally upon urgent recommendation to publish this latter report as soon after receipt as practicable. The outcome of the endeavor thus to maintain a close connection between the Board and the banks, and

between the latter and their members, as well as between the System as a whole and the public was the authorization to begin the publication of the Federal Reserve Bulletin, which for eight years, from 1914 to 1922, it was one of my duties to edit and publish. Fundamentally the Bulletin was intended as a frank, truthful and complete exposition of banking conditions, not only those narrowly pertinent to the System itself, but also those belonging to the entire group of banks included within the scope of the System. In order to make such reports at all complete, it was essential to install a weekly report of member bank conditions, and since this could not be obtained from every bank in the System promptly enough to permit immediate publication, it was resolved to print it as representing the status of banks in one hundred selected and representative cities, publishing the same by statement to the newspapers first, pending the arrival of the data for the monthly issuance of the Federal Reserve Bulletin. Thus was completed the general system of reports which had been provided for in the original Federal Reserve Act, and in the report of the Technical Committee on Organization which had recommended most of the actual steps afterward agreed on. A foundation has thus been laid for regular banking self-analysis, although the use of the data spreads but slowly.

OPPOSITION TO SCIENTIFIC STUDY

It is instructive at this point to stop for a brief review of the difficulties encountered in making even the present amount of advance. First of all, let me recall the fact that the maintenance of the Federal Reserve Bulletin in any form was, during the first eight years of its life, a constantly debated question, a certain party in the Board being usually opposed to its maintenance and the governor of the Board at one time demanding its discontinuance. The Bulletin itself had been initiated without the prior consent of the then Secretary of the Treasury, and the first comment of that official when an issue of the Bulletin was laid before him was simply, "This Government ain't going into the newspaper business." The point of view thus revealed was somewhat modified, not so many months later, when the Wilson Administration began the publication of a daily war newspaper, the *Official Bulletin*. The Federal Reserve Bulletin, in fact, was never a "newspaper" or "magazine," or anything approximating those types of publication, and the effort of some to disestablish it as a statistical summary of progress was due primarily to the inconvenience to sundry interests arising out of the wide publication of the actual facts in the development of our banking system and of credit under it. It is the same opposition to real fact knowledge and to fact analysis, which

usually manifests itself in the minds of political administrators, that has always stood in the way of every effort to develop business and credit statistics to their full possibilities under the Federal Reserve System.

BUSINESS KNOWLEDGE NEEDED

As the Federal Reserve System was gradually evolved, it became plain that, as had been found in all other Central banking systems, the successful supervision and direction of credit from a central source implied profound knowledge of business conditions. And yet a survey of experience during the past thirty years shows convincingly how ragged, incomplete and unsatisfactory our knowledge of business statistics has been. Take, for example, the reports of the commercial agencies which for many years were the staple of information in this field. These reports were founded upon a very few fact items—chiefly the volume of clearings reported by clearing houses, the number and volume of business failures, new incorporations, and new issues of securities (and even these two latter items of information upon only a partial and incomplete basis). Evidently there was good reason for the practice of many business reporting agencies in making their weekly or monthly surveys studies in psychology rather than in facts. A typical account of business at any given season of the year, such as "crop-moving" time, or at the holiday season, would usually be made up chiefly of generalities such as, "The volume of trade this year is proving very satisfactory; goods are moving freely into the hands of consumers. Employment is satisfactory and demand for labor good." There was but little of real knowledge. Coupling this lack of fact material with the always existent indisposition to "hurt business" or to speak pessimistically of the prospects, it was undeniably true that these reports not only came a long time after the situations to which they related had developed, but that they were only very remote approximations to the facts in the case. Altogether, therefore, it was obviously the case, and still is true, though in less degree, that only an insufficient structure of business statistics was available as a support for wise banking action.

Greatly to the credit of the Federal Reserve Board is the fact that, once this situation had been duly perceived, it had the courage to embark upon the development of as inclusive a set of business statistics bearing upon credit and credit conditions as circumstances would permit. It was perceived from the very outset that it would be out of the question to obtain promptly for current use a set of data actually reflecting details of the situation in a large industry. The effort made, therefore, was to obtain representative statistics calculated to indicate

the status of affairs in specified businesses which were of representative character. For example, there being at the time no reliable data concerning the volume of retail trade, it was sought to build up a retail trade index, which was originated and capably developed by Dr. W. H. Steiner, then Assistant Director of the Division of Research. Effort was further made by the same officer to obtain reliable production statistics in representative industries, both these and the retail trade figures being obtained from selected establishments whose doings might be regarded as constituing a fairly stable measure of the conditions throughout the industry, just as the unfilled orders of the U. S. Steel Corporation had long been regarded as indicative of the steel trade situation and outlook. Again endeavor was made to obtain a means of regularly measuring the volume of our exports in terms of physical units, the result being export and import trade index, originally computed and set on foot by Mr. F. W. Jones, a member of the research staff of the Division of Analysis and Research. A shipping index designed to furnish a careful computation of the proportion of shipping freights received and paid as a factor in international trade was also prepared by another member, Mr. H. W. Van Pelt, and received the unusual compliment of being copied by British government authorities who at once recognized its advantages. In like manner indexes of foreign exchange rates based upon quotations selected upon a uniform basis and amalgamated after a scientific principle were developed for the purpose of indicating the purchasing power of the dollar abroad. Commodity price statistics also received extensive attention, the endeavor being to do what had not previously been attempted, build up with the aid of foreign authorities price indexes for the principal trading countries based upon commodities of local production primarily, chosen with a view to the paralleling of the commodities chosen in other countries as a basis for such index work. Thus indexes for Great Britain, Canada, France and Japan were worked out under the direction of Miss K. M. Snodgrass, and publication thereof begun. It would be possible to extend the list of these statistics much further and to indicate more in detail the character of the studies of business statistics which were thus set on foot, but enough has been said to show the general scope of the new investigations that were authorized. Distinct aid was derived from the fact that work along parallel lines was being done especially in the commodity field by others, particularly by the Harvard Committee on Economic Research, while valuable studies had been made during the war, helped by the unusually perfect conditions of control and supervision of industry which then existed, under the direction of the War Industries Board and other

bodies. Although these were for the most part discontinued at the close of the war, the work done remained as an index of possibilities, and as a nucleus around which to build subsequent studies. The war undoubtedly gave a material impetus to the study of business statistics. In the credit field, the Board, with the consent of the American Bankers' Association, took over from the latter the task of compiling inclusive figures as to bank debits, intended eventually to supersede the imperfect clearing house figures whose defects had so long been obvious; and I was successful in broadening and perfecting the basis of these valuable series.

OPPOSITION; ITS ORIGIN

I believe that there is no more instructive study in the recent history of business analysis than the attitude which has been adopted with respect to these advances in statistical knowledge by our banking system itself, by the business public, and lastly by the government. The first "reaction" to the new plans naturally came from the reserve banks and may be regarded as undoubtedly representing the reflected attitude of the local banking and business community in each center. This attitude was unmistakably hostile. Not only did some of the banks present constant argument against the attempt to gather sundry of the statistics which they were requested to obtain, but they seemed to regard the entire undertaking as a purely academic—and hence undesirable—"frill" upon the administration of the banking system. At one of the Western banks, it was reported to me by an assistant Federal Reserve agent who had at length been placed in charge there after a year or two of postponement and disinclination to coöperate, that not only had nothing been done, but that the local banking and business community had been in some way indoctrinated with distaste for the whole undertaking and were inclined to subject it to ridicule. At other banks, complaint was at first made of the great cost involved in obtaining the data, although it was never possible to obtain accurate comparative statistics as to what this cost might be. So-called statisticians at still other banks devoted their time to complaints of the fact that the data thus to be obtained could never be accurate, as measured by census standards, and they produced lengthy disquisitions designed to substantiate their opinions on the subject. The first few months of the new work were of the most disheartening character, not because of lack of progress but because of the entire lack of appreciation of any such undertaking.

As to the business community, the situation was much less uniform, but contained large elements of discouragement of its own sort. Busi-

ness men in general have long felt the annoyance produced by the constant sending of inquiries or "questionnaires" from all sorts of government offices, including tax authorities, trade commission investigators, commercial experts, census inquirers, and a good many others. They had found that it was, at least occasionally, true that data furnished by them in confidence would be used for the purpose of initiating prosecutions under the anti-trust law. They naturally disliked the trouble and labor involved in making the statistical compilations, always costly and troublesome at the best, and often involving a process of self-analysis which showed that their own records were not satisfactory and up to date, and that they ought in their own interest to install new systems.

Contact with the business community was established either through trade associations or through representative firms. Of the two, the trade associations were found to be in some respects much the more promising field, since their officers usually appreciated the desirability of the data sought. Due, however, to the fact that such officers were frequently regarded by the members of the associations, as little more than semi-clerical functionaries, the success attained in getting data from them was for a long time very limited. As for the individual business houses, it was of course necessary to overcome their objections and hesitation, based upon the grounds already outlined, and to persuade them that they were undertaking a service to the community which would almost immediately be reflected in real commercial benefit to themselves. It became apparent that the general condition of indifference and reluctance to furnish such data was so widespread that some years of "missionary work," with attendance at trade associations, and with explanations fully made to individual business men would be necessary, if actual results were to be accomplished. And yet, it must be said that the few years since this labor was begun have been productive of very decided response on the part of the business community. Experience demonstrated that it is entirely possible to obtain the coöperation of business men in the effort to build up extensive series of current business statistics.

GOVERNMENT "OFFICE-POLITICS"

One reason why more progress has not been already made in this connection is afforded by the unexpected political obstacles that have presented themselves, but another and even more important difficulty by the singular growth of jealousy and interference on the part of various branches of the government. It was an astonishing thing that a department of the government, under the administration which came

into office in 1921, should have sought to duplicate and probably destroy the activities of the Federal Reserve System which were just then reaching a stage of fruition as regards business analysis and statistical study. And yet this was the case. Not only was there a deliberate attempt to undercut the work that had been done by the Federal Reserve Board and even at times to discredit it, but the Board was unofficially requested to turn over data both domestic and foreign. These were then used by this department, often without the slightest recognition or credit as to source, in the development of its own business statistics. Efforts to anticipate the monthly reports that were being made by the Board in order to get more "publicity" was an attendant circumstance, the whole outcome being naturally to render the entire reporting enterprise ridiculous, to increase the irritation already felt by those business men who were indisposed to take much trouble in reporting, and to give opportunities for discrepancies which were made much of by critics, and actually furnished a real basis for complaint in view of the fact that two governmental offices were reporting slightly different conclusions, and hence affecting prices and productive conditions.

The politics of the situation came to the front in another way. Although the Federal Reserve System had been conducted on a basis of substantial economy, the war and its subsequent developments had naturally increased the expense of the System tremendously. When complaint in Congress and elsewhere began to develop with respect to this basis of expenditure, it was a grotesque and amusing feature of the case that not a few inside the System, as well as without it, including some in Congress, should endeavor to place the responsibility for high costs partly upon the scientific work of the Board. At that time (1921) the Board itself was spending in the neighborhood of $50,000 a year for salaries of those engaged in its business statistics and research work, while the banks of the System (the twelve Federal Reserve banks) were probably spending perhaps $250,000 more, most of it at some two or three institutions. Bearing in mind that the aggregate expenses of the System were then in the neighborhood of $35,000,000 or more (they were a little under $30,000,000 for 1923), the effort to place responsibility upon "statistics" as the cause of high costs seemed somewhat amusing. And yet this effort was the assigned reason for materially reducing the scope of the work that was being done. Taken in conjunction with the side attack involved in the efforts of the government department already referred to, the joint effect was sufficient greatly to undermine and limit the scope of the endeavor and eventually to reduce it to a basis of relative inefficiency. It should

be noted, too, that at this juncture came a determined effort from within the Federal Reserve Board itself to disestablish all business statistics and all research work. Curiously enough, the reserve banks, some of which had been so slow and so unwilling to introduce the work at all in the first instance, had in the meantime come to recognize the necessity of it, being influenced in no small degree by the fact that their own communities had responded so strongly and with so nearly unanimous a voice in its favor. As they had been at the outset a factor largely responsible for the retardation of growth and progress, they now became a factor tending to stabilize and preserve what had already been accomplished, and to their efforts in large measure must be ascribed the fact that the whole system of business and credit statistics was not entirely done away with.

ATTACK OF DEPARTMENT OF JUSTICE

It would seem that the effort to provide a scientific basis for credit and for business operations generally, through the introduction of a compact system of reliable business statistics, had had enough obstacles to overcome, but conditions indicated otherwise. The Attorney-General of the United States, early in 1922, began to take definite position to the effect that the compilation of statistics of production and prices through trade associations or through the joint effort of business men therein organized, was an activity distasteful to the government as guided by the anti-trust laws, and he therefore in various opinions took occasion to direct the discontinuance of such statistics. In various prosecutions which were set on foot against business men, the fact that there had been a scientific effort to obtain reliable data as to production and prices, although not accompanied by any evidence whatever of combination or of "price fixing," was cited as contributory evidence tending to substantiate the notion of violation of the anti-trust laws. Notwithstanding efforts on the part of other officers of the government to bring about a more reasonable position on the part of the Attorney-General, they were never completely successful, and the position of the Department of Justice continued up to the winter of 1923–4 to be a hindrance of the first rank in the way of obtaining responsible data. If it be true that capital is traditionally timid, it is also true that business is super-timid, and the prospect of attack on the part of the government, or rather of one branch of the government because individuals or groups had sought to do what another branch had requested and even urged, was sufficient to disgust many, especially of those who had been weak-kneed from the beginning, with the whole undertaking, which aimed to create a statistical basis for business.

PROGRESS AND MATHEMATICS

Nevertheless, decided progress has been made, and the decade which closed with 1923 will probably always stand as a period of genuine advance in this field of statistics. It is unfortunate that better use has not been made of the new data by those through whose hands they have passed. They have undoubtedly been inadequate in their treatment of materials. One of the things which has tended to obscure the usefulness and to limit the general acceptability of our new business statistics has been the ever-present disinclination on the part of statisticians to add to and complete the fact basis upon which they rest. Instead of such effort for completion, there has been a distinct effort to cover up defects. It is always easier to suggest so-called "mathematical" methods, or to compute "normal" bases, or otherwise to give an appearance of abstruseness and "scientific" quality than it is to do the real spade-work necessary to the cultivation of a field in which business and political obstacles are added to the natural hindrances which would stand in the way of success even under the most favorable circumstances. Unfortunate, certainly, it is that some of the reserve banking statistical organizations, as well as others, unwilling to do any real work in gathering fresh data under difficulties have chosen to occupy their time in this way. It is rather poor consolation that many of those who at first decried such series as those on retail and wholesale trade, refused to undertake the hard work of collecting them, and spent their time largely in writing letters to demonstrate their impracticability now devote themselves, since the system of statistics is an established fact, to mathematical computations designed to refine upon the data which were at first rejected as worthless or unattainable. The fact that some pride themselves upon the results obtained, ignoring the tenor of the copious files of their earlier correspondence, adds a touch of spice to the situation and illustrates how greatly the "human equation" must be allowed for even in the driest of statistical undertakings.

SOME ELEMENTS OF GAIN

That, in spite of these various difficulties, real progress, as already said, has been made, must be attributed directly to the genuine and intrinsic merit of the undertaking. I believe that in summing up the elements of advance in business statistics during the past ten years we may fairly include the influence that has been exerted by the statistics compiled under the influence of our Reserve System upon other statistical organizations, research bureaus, and business reporting services, and may summarize the main developments of the movement of which it has been a part somewhat as follows:

1. A partial abandonment of the old-style "psychological" business report in which the terms good, bad, poor, etc., were used to describe business conditions without any uniform conception of their meaning. In place of this old-style type of report there is now found the modern style of business analysis based on facts and to a very large extent presenting its data in the percentile index form.

2. A recognition of the value to the individual business house of a knowledge of the trend of affairs in the trade or business as shown by representative statistics relating to production, distribution, credit, and other matters of the same sort.

3. A partial admission that our knowledge of business facts must be very greatly improved or enlarged before very much service can be rendered by elaborate mathematical computatior designed to supply missing "terms," or to refine upon data which are not sufficiently complete to warrant such refining.

4. Far better knowledge of the current position of credit as evidenced by banking reports of condition and far better analysis of the relation of bank credit in the aggregate to business demand at any given moment.

5. A large development of the theory of banking supervision or control and of the importance of generalized credit knowledge in making an appraisal of the credit position of the individual business.

6. The laying of a foundation for some rational amendment of the anti-trust laws when the time comes that a moderately reasonable state of public opinion will permit sober review of the problem.

7. Initiation of genuine business statistical research on the part of a number of the larger business institutions of the country.

A GENERALIZED LAW OF ERROR

By Paul R. Rider, *Washington University*

In an article on "First and Second Laws of Error" [1] Professor E. B. Wilson has shown by his analysis of certain data that they give evidence of being distributed according to Laplace's first law of error, which states that the frequency of an error or deviation can be expressed as an exponential function of the absolute value of the error, rather than according to Laplace's second law—called the normal law or Gauss's law —which states that the frequency is an exponential function of the square of the error. As it hardly seems reasonable to suppose that

[1] Jour. Amer. Stat. Assn., Sept. 1923.

there is a sharp line of demarcation between data that follow the first law and those that follow the second, the writer would like to suggest the more general law that the frequency of an error can be expressed as an exponential function of the mth power ($m>0$) of the absolute value of the error. Stated in mathematical symbols this law says that if y is the frequency of an error x then for a distribution of N items we have

$$y=\frac{N\,m\,k^{1/m}}{2\,\Gamma\,(1/m)}e^{-k\,|x|^{\,m}}. \tag{1}$$

The error or deviation x might be measured from any central value; in this paper it will be measured from the median.

Our generalized law reduces for $m=1$ to Laplace's first law

$$y=\frac{N\,k}{2}\mathrm{e}^{-k\,|x|}, \tag{2}$$

and for $m=2$ to the normal law

$$y=\frac{N}{\sigma\sqrt{2\pi}}e^{-x^2/2\sigma^2},\ \sigma=\frac{1}{\sqrt{2k}}. \tag{3}$$

(In the latter case we assume that mean and median coincide.)

For the value of y defined by (1) we have

$$\int_{-\infty}^{\infty} y\,dx=N, \tag{4}$$

$$\int_{-\infty}^{\infty} |\,x\,|\,y\,dx=\frac{N\,\Gamma(2/m)}{k^{1/m}\Gamma(1/m)}, \tag{5}$$

$$\int_{-\infty}^{\infty} x^2y\,dx=\frac{N\,\Gamma(3/m)}{k^{2/m}\Gamma(1/m)}. \tag{6}$$

If we wish to fit a curve of the type (1) to an observed distribution we employ the moments $S_1=\Sigma\,|x|/N$, $S_2=\sqrt{\Sigma x^2/N}$ of the observed distribution and set

$$\frac{\Gamma(2/m)}{k^{1/m}\Gamma(1/m)}=S_1,\ \frac{1}{k^{1/m}}\sqrt{\frac{\Gamma(3/m)}{\Gamma(1/m)}}=S_2. \tag{7}$$

Dividing the second of the preceding equations by the first, we get

$$\frac{\sqrt{\Gamma\,(3/m)\,\Gamma\,(1/m)}}{\Gamma\,(2/m)}=\frac{S_2}{S_1}. \tag{8}$$

In order to find m let us set $r=1/m$ and take logarithms of both sides of (8). We obtain the equation

$$\log \Gamma(3r)+\log \Gamma(r)-2\log \Gamma(2r)=2\log (S_2/S_1), \qquad (9)$$

which is not difficult to solve for r if one has at hand a good table of logarithms of the gamma function.

Having found m by taking the reciprocal of r, we can obtain k from either of equations (7). (In the case of the normal curve we use the second.) In practice it would be well to solve both of equations (7) for k in order to check.

As an application we shall analyze the data considered by Professor Wilson and earlier by Professor W. L. Crum.[1] These data are shown in Table I.

TABLE I

FREQUENCIES OF DEVIATIONS FROM THE MEDIANS

Deviation	Frequency	Deviation	Frequency	Deviation	Frequency
* Over −30	2	−11	6	6	13
−30	1	−10	3	7	8
−29	1	− 9	5	8	6
−28	1	− 8	11	9	5
−24	1	− 7	6	10	2
−23	1	− 6	23	11	4
−22	1	− 5	10	12	3
−21	2	− 4	13	13	1
−20	1	− 3	19	14	2
−19	2	− 2	9	15	1
−18	2	− 1	11	16	1
−17	2	0	28	17	1
−16	2	1	22	18	2
−15	1	2	22	23	1
−14	3	3	13	24	1
−13	6	4	19	28	1
−12	3	5	13	† Over 30	7

* −32, −37.

† 34, 35, 35, 41, 41, 42, 45.

Let us first consider separately the negative deviations. We find $S_1=1225/162=7.5617$, $S_2^2=17341/162=107.043210$, $S_2=10.346$, $m=1.16$, $k=0.077$. Thus the equation of our frequency curve is

$$y=324\times 0.05776e^{-.077|x|^{1.16}}=18.714e^{-.077|x|^{1.16}}. \qquad (10)$$

The theoretical frequencies are found by evaluating the integral $18.714\int_{|x|-3}^{|x|+3} e^{-.077|x|^{1.16}}dx$, a six-unit interval being employed. The values in the present problem were obtained by means of a series when $x=3$ and by means of Simpson's rule for other values of x.

[1] "The Use of the Median in Determining Seasonal Variation." JOUR. AMER. STAT. ASSN., Mar., 1923.

The theoretical frequencies are compared with the observed frequencies and with the theoretical frequencies according to Laplace's first law in Table II. It is seen that the fit is somewhat better for the generalized law.

TABLE II

x	n	n_1	n_2	x	n	n_1	n_2
− 3	87.5	90.5	86	3	109.5	108	106
− 9	44	40	43	9	33	36	30
−15	16.5	18	19.4	15	8.5	11.5	12.6
−21	8.5	8.5	8.7	21	2.5	4.0	5.9
−27	3	4	3.4	27	1.5	1.3	3.0
−33	1.5	1.7	1.4	33	3	0.45	1.64
−39	1	0.75	0.53	39	2.5	0.15	0.93
				45	1.5	.....	0.54

x is the deviation from the median.

n is the observed frequency.

n_1 is the theoretical frequency according to Laplace's first law (values calculated by Professor Wilson).

n_2 is the theoretical frequency according to the generalized law.

For the positive deviations we find the theoretical curve

$$y = 46.873\ e^{-.57|x|^{.63}}. \tag{11}$$

Theoretical and observed frequencies are compared in Table II. Apparently Laplace's first law gives a better fit in this case than the more general law. However, it may be remarked that testing the two theoretical distributions for goodness of fit shows that on the whole the generalized law is better.

It would perhaps seldom be worth while to fit a set of data with a curve of the suggested general type any more closely than to find the best first or second Laplacian curve of error. That is, it would doubtless be sufficiently accurate in most instances to use $m=1$ or $m=2$ in equation (1), especially since the methods of fitting curves for these special values of m have been well developed and are comparatively simple. Nevertheless, it seems quite possible that certain data, when subjected to a refined analysis, may be found to conform more closely to the general law proposed where m has some other value than 1 or 2. The negative deviations in the data studied support this contention.

DEATH RATES FROM PUERPERAL SEPTICEMIA AND THEIR VARIABILITY

By G. E. Harmon, *From the Department of Hygiene and Bacteriology, School of Medicine of Western Reserve University*

The present inquiry was suggested by the variations in the rates from puerperal septicemia observed during a previous statistical study of its course in certain divisions of the United States.[1] In that investigation, because of lack of accurate information concerning the number of living births and stillbirths for most sections of the United States over any long period, it was impossible to relate the number of deaths from puerperal septicemia to the sum of the living births and stillbirths as suggested by Howard.[2] The expedient was tried of relating the deaths from puerperal septicemia to the number of married females between the ages of 15 and 44. This involved the calculating of a number of rates for certain states expressed as the number of deaths from puerperal septicemia per 100,000 married females, age 15–44.

Rates were calculated only for those states which have been in the United States registration area for deaths since 1900, namely, Connecticut, Indiana, Maine, Massachusetts, Michigan, New Hampshire, New Jersey, New York, Rhode Island and Vermont; and four states of the five admitted in 1906 which have remained permanently in the registration area, namely California, Colorado, Maryland and Pennsylvania.

Table I presents the frequency distribution of the rates for the years specified according to the method of analysis just noted, and gives some indication as to their variability. A consideration of the table clearly shows that the yearly rates are variable. This is particularly evident for 1900, 1904, 1906, 1908 and 1918. For these years the highest rates are from three to six times the lowest. The rates for the years 1903, 1915 and 1921 show less variability, and in fact are quite stable when compared with those for the years just mentioned.

In Table II there are presented certain measures of variation—the mean, the standard deviation and the coefficient of variation—of the rates experienced by the states of the group previously indicated. The means were calculated from the various rates obtained and not by using the mid-values of the class intervals and frequencies of Table I. The probable error of the means and standard deviations was not

[1] Harmon, G. E. *American Journal of Public Health*, Vol. 14, p. 214. 1924.
[2] Howard, W. T. *American Journal of Hygiene*, Vol. 1, p. 197. 1921.

TABLE I

FREQUENCY DISTRIBUTION OF THE DEATH RATES FROM PUERPERAL SEPTICEMIA FOR CERTAIN STATES IN THE DEATH REGISTRATION AREA OF THE UNITED STATES

Deaths per 100,000 married females (age 15–44)	1900	01	02	03	04	05	06*	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	Total
0–9	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
10–19	1	..	..	..	..	..	1	..	1	..	..	..	1	..	..	..	1	..	..	..	..	..	5
20–29	2	4	4	2	1	1	..	2	..	..	..	1	1	1	2	1	2	..	3	2	2	..	31
30–39	3	3	2	5	2	3	5	3	4	5	4	3	3	4	2	6	3	5	4	7	4	5	85
40–49	2	1	2	2	4	4	4	4	5	3	5	5	6	3	5	5	4	4	4	4	5	4	85
50–59	..	1	2	1	1	1	3	3	2	4	2	2	2	4	4	2	4	4	1	1	2	5	51
60–69	2	1	..	..	1	1	..	1	1	2	2	2	1	2	1	..	..	1	1	..	1	..	20
70–79	..	..	..	..	..	..	..	..	1	..	1	1	..	..	..	..	..	..	..	..	..	..	3
80–89	..	..	..	..	1	..	1	1	..	..	..	..	..	..	..	..	..	..	1	..	..	..	4
Total	10	10	10	10	10	10	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	284

* Five states admitted to the registration area for deaths in 1906. One not included in this study because it was later dropped.

calculated because of the small number of observations included. Under such circumstances, according to Yule,[1] the probable error is of doubtful value but the standard deviation retains its significance as a measure of dispersion. The coefficient of variation is included to relate the variation to the mean, but it should be understood that the significance of this constant is of doubtful value in comparing the relative variability of different series of death rates.

TABLE II

MEASURES OF VARIATION OF THE PUERPERAL SEPTICEMIA RATES INCLUDED IN TABLE I

	Mean	Standard deviation	Coefficient of variation		Mean	Standard deviation	Coefficient of variation
1900	40.4	15.0	37.1	1911	46.9	12.4	26.4
1901	36.7	13.5	36.8	1912	41.0	10.6	25.8
1902	37.6	12.1	32.0	1913	45.7	11.3	24.7
1903	36.5	8.0	21.9	1914	44.5	9.6	21.5
1904	46.2	15.0	32.5	1915	40.2	8.2	20.4
1905	43.2	11.8	27.4	1916	41.5	13.2	31.8
1906	43.5	14.9	34.2	1917	45.1	10.1	22.4
1907	46.2	14.4	31.2	1918	41.8	15.7	37.6
1908	46.1	15.0	32.5	1919	37.2	7.5	20.2
1909	46.5	11.5	24.7	1920	42.0	11.0	26.2
1910	47.9	11.1	23.2	1921	44.5	8.4	18.9

A study of Table II will show that there was no tendency for the rates from puerperal septicemia to decline during the period considered, although the means for the years at the middle of the period are larger than those for the years at either end. Judging by the stand-

[1] G. Udney Yule. *An Introduction to the Theory of Statistics*, p. 311.

ard deviation, a constant that measures the absolute amount of dispersion, the rates from puerperal septicemia are quite variable, and over the entire period as a whole exhibit no significant tendency to become more stable.

Since the establishment of the United States registration area for births in 1915, birth statistics have been available for a number of states. With this information at hand it is possible to calculate the number of deaths from puerperal septicemia per 10,000 births, a method more nearly approximating the ideal one suggested by Howard [1] than that employed in the first part of this paper. In the remainder of the present discussion we shall calculate the rates for each of three classifications of the population of a state, namely, portion living in cities of 100,000 and over, portion living in cities of 10,000 to 100,000 and portion living in communities of less than 10,000. The total number of sections of state population for each division of the classification of course varies from year to year depending upon the number of states in the registration area for births. Rates were also calculated for each city of 100,000 and over situated in the registration area for births taken as a whole. The number of these cities also varies from year to year. Table III presents the frequency distribution of the rates for the various sections of state population and for cities for the indicated years, and gives some idea as to the size of the rates which have been experienced and their variability.

A study of this table will clearly show that the rates have been quite variable, those for cities of 100,000 and over and for sections of state population living in cities of 10,000 to 100,000 being the most variable and the highest.

In Table IV there are presented certain measures of variation of the rates from puerperal septicemia for the various distributions of Table III. The means have been calculated from the original data as described under Table II and, as was mentioned in connection with that table, the meaning of "coefficient of correlation" is not clear and needs therefore to be interpreted with care.

This table indicates that the rates are variable and show no significant decline for the period considered. In those sections of population made up of those living in cities of from 10,000 to 100,000 the rates have been higher and most variable, while those sections of population composed of those living in communities of 10,000 or less have the lowest rates and exhibit the least variability when this is measured in absolute units.

[1] Howard, W. T. *American Journal of Hygiene*, Vol. 1, p. 197. 1921.

TABLE III

FREQUENCY DISTRIBUTION OF THE DEATH RATES FROM PUERPERAL SEPTICEMIA FOR CERTAIN STATES, SECTIONS OF STATE POPULATION AND CITIES IN THE BIRTH REGISTRATION AREA OF THE UNITED STATES

Deaths per 10,000 living births	States								Sections of state population composed of the total number living in cities of 100,000 and over								Sections of state population composed of the total number living in cities of 10,000 to 100,000								Sections of state population composed of the total number living in communities of less than 10,000								Cities of 100,000 population and over							
	1915	16	17	18	19	20	21	Total	1915	16	17	18	19	20	21	Total	1915	16	17	18	19	20	21	Total	1915	16	17	18	19	20	21	Total	1915	16	17	18	19	20	21	Total
0–4	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	..	1	1	..	..	..	..	..	..	1	1	..	..	..	..	..	1	..	1
5–9	..	1	..	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	1	1	2	2	2	1	2	1	11	1	1	..	1	..	1	..	4
10–14	..	1	..	3	..	1	..	5	..	1	..	1	1	..	..	3	2	..	..	..	..	..	..	2	1	3	3	3	3	1	2	16	..	1	..	1	3	1	1	7
15–19	6	1	2	2	7	3	3	24	1	..	1	2	1	..	..	5	1	2	1	2	5	4	3	18	4	4	3	6	9	10	7	43	6	2	5	8	4	5	5	35
20–24	1	5	8	6	6	9	7	42	2	1	3	1	3	2	1	13	2	3	3	3	1	2	1	15	3	2	8	6	6	4	11	40	2	3	4	..	7	7	5	28
25–29	3	1	4	5	5	5	11	34	1	1	1	3	4	6	4	20	2	1	1	2	2	1	1	10	..	..	3	1	2	3	5	14	6	..	5	6	6	8	6	37
30–34	..	2	4	2	2	4	5	19	3	1	4	3	1	5	5	22	3	2	3	2	1	2	6	19	..	..	1	1	1	3	..	6	2	4	6	6	4	6	10	38
35–39	..	..	2	..	1	..	1	4	..	3	1	2	2	1	4	13	..	1	3	4	7	3	4	22	..	..	..	1	..	..	..	1	..	5	3	3	3	8	11	33
40–44	..	..	..	2	1	1	..	4	..	1	3	1	..	3	3	11	..	..	..	1	1	4	5	11	..	..	..	..	..	..	1	1	2	5	4	3	2	3	5	24
45–49	..	..	..	..	..	..	..	..	..	..	1	1	..	..	2	4	..	..	5	1	1	2	1	10	..	..	..	..	..	..	..	..	1	..	2	2	1	2	2	10
50–54	..	..	..	..	..	..	..	..	..	..	..	..	3	1	1	5	..	1	1	2	..	2	1	7	..	..	..	..	..	..	..	..	..	..	..	1	5	1	3	10
55–59	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	1	2	2	2	8	..	..	..	..	..	..	..	..	..	..	1	1	..	2	1	5
60–64	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	1	..	2	4	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	1
65–69	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	1	..	..	..	1	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	3	3
70–74	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	1	..	..	..	..	..	..	..	..	..	..	1	..	..	1	..	2
75–79	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
80–85	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
85–89	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	1
150–154	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Total ..	10	11	20	20	22	23	27	133	7	8	14	14	15	18	21	97	10	11	20	20	22	23	26	132	10	11	20	20	22	23	27	133	20	21	32	32	35	46	53	239

TABLE IV

MEASURES OF VARIATION OF THE PUEPERAL SEPTICEMIA RATES INCLUDED IN TABLE III

		Mean	Standard deviation	Coefficient of variation
States	1915	20.6	3.87	18.7
"	1916	21.7	6.64	30.6
"	1917	26.3	5.52	20.9
"	1918	24.2	8.12	33.5
"	1919	24.6	6.38	25.9
"	1920	25.5	6.38	25.0
"	1921	26.8	4.75	17.7
Sections of state population composed of the total number living in cities of 100,000 and over	1915	26.4	5.31	20.1
"	1916	30.9	9.60	31.0
"	1917	32.3	9.46	29.3
"	1918	29.9	9.44	31.5
"	1919	31.6	11.57	36.6
"	1920	33.7	7.77	23.0
"	1921	37.3	10.27	27.5
Sections of state population composed of the total number living in cities of 10,000 to 100,000	1915	23.7	6.94	29.2
"	1916	26.2	12.75	48.7
"	1917	40.4	14.24	35.4
"	1918	34.8	15.10	43.4
"	1919	39.9	27.71	69.4
"	1920	38.0	14.32	37.6
"	1921	38.4	12.24	31.9
Sections of state population composed of the total number living in communities of less than 10,000	1915	16.1	5.55	34.4
"	1916	15.1	5.32	35.2
"	1917	20.4	6.69	32.8
"	1918	19.6	7.33	37.4
"	1919	19.4	5.54	28.5
"	1920	20.6	6.90	33.5
"	1921	21.3	6.96	32.6
Cities of 100,000 population and over	1915	26.0	10.41	40.0
"	1916	30.5	10.22	33.5
"	1917	33.4	13.38	40.0
"	1918	30.2	12.03	39.8
"	1919	30.7	11.81	38.4
"	1920	31.5	12.96	41.2
"	1921	36.2	14.17	39.2

CONCLUSIONS

The rates from puerperal septicemia, whether expressed as the number of deaths from this cause per 100,000 married females or per 10,000 births, exhibit considerable variation.

As judged from the means for the various years, the rates show no significant decline and there seems to be no tendency for them to become more stable or less scattered.

Sections of state population composed of all those living in cities of from 10,000 to 100,000 have high rates and the rates exhibit marked variability.

Sections of state population composed of those living in communi-

ties of less than 10,000 have the lowest rates and exhibit the least variability.

Granted that a high rate and large absolute variability indicate a favorable field for public health activities, organized efforts directed toward the prevention of puerperal septicemia would apparently promise quicker results in cities of 10,000 population and over rather than in smaller communities.

COTTON STATISTICS FROM THE DEPARTMENT OF COMMERCE

The recent marked drop in the price of raw cotton, entailing or threatening heavy losses to American owners of that staple, focussed attention especially in the South upon the cotton statistics issued periodically by the Department of Commerce. It is the practice of that department soon after the close of the cotton year on August 1 to issue a statement showing the estimated amount of cotton carried over from the preceding year, the amount ginned and the amount imported, these summing to the year's supply. On the other side appear the amount used, the amount exported, the amount destroyed and the amount carried over to the following year, these summing to the year's consumption. The two sides of the account do not and should not be expected to balance closely; indeed, if they should, it would arouse suspicion that the figures had been forced. It has been the custom, under what is perhaps a questionable procedure in a case of this sort, to introduce in the statement an item, "to balance." In the statement of August, 1923, this item, "to balance distribution 579,504," or 4.3 per cent of the year's supply was larger, I believe, than ever before and some months later when the discussion became acute it was on this item that attention was fastened. At a conference between Secretary Hoover and several senators from cotton-growing states it was decided to appoint a committee of expert statisticians who would study the situation and make suggestions for its improvement. Two southerners familiar with the production of cotton, B. W. Kilgore and D. C. Roper, were named and two notherners familiar with statistical problems and methods, W. S. Rossiter and W. F. Willcox. To these was added Dr. L. I. Dublin, president of the American Statistical Association. Later, on the ground that his connection years before with the system of cotton statistics built up by the Census Bureau might expose him or the committee to criticism, Mr. Roper withdrew from the committee but as an assessor he continued to give it the benefit of his special knowledge and experience. A quick report was desired and accordingly the committee finished its work and reported within less than a month. The report prefaced by a brief press summary was as follows:

W. F. W.

PRESS REPORT

The results of the efforts of the investigation of Mr. Hoover's Cotton Committee is to make several important changes in the figures of the report in dispute, namely, that of August 18, 1923. The leading item of contention was 579,504

bales which appeared in that report under the head of "To balance distribution." In the revised report of the committee this item shows as 355,868 bales with the following explanation:

> "It is impossible to determine just how much of this may be due to further duplication in mill and warehouse stocks or to the understatement of stocks and carry-over at the beginning of the year. The committee recommends that in future statements of the Census Bureau no attempt be made to balance the figures of supply and distribution. The Census Bureau should issue figures which are the result of enumerations, avoiding estimates."

REPORT

WASHINGTON, D. C.,
March 25, 1924.

HONORABLE HERBERT HOOVER,
Secretary of Commerce,
Washington, D. C.

SIR:

In calling into existence the committee which signs this report you outlined its purposes as follows:

"A contention has arisen over the methods of the Census Bureau in cotton statistics and also over the accuracy of a cotton statement issued last August. There is also a question as to whether the Bureau has sufficient authority to secure adequate information on the subject. With a view to having independent advice on the entire matter I have arranged with the Senators from the cotton states that a committee should be appointed of leading statisticians on commerce to examine into the whole matter."

The work of the committee, therefore, falls under these heads:

(1) To study "the methods of the Census Bureau in cotton statistics."

(2) To determine "the accuracy of a cotton statement issued last August."

The figures of this report are as follows:

			(*Bales*)
SUPPLY			
Aggregate			13,610,218
Stocks August 1, 1922, total			2,831,553
In consuming establishments, total		1,218,388	
In cotton-growing states	531,312		
In all other states	687,076		
In public storage and at compresses, total		1,488,165	
In cotton-growing states	1,123,101		
In all other states	365,064		
Elsewhere (estimated)		125,000	
Ginnings			9,729,306
Imported August 1 to July 31, 1923			469,954
To balance distribution			579,504
DISTRIBUTION			
Aggregate			13,610,218
Consumed August 1, 1922 to July 31, 1923, total			6,664,710
In cotton-growing states		4,248,525	
In all other states		2,416,185	
Exported August 1, 1922 to July 31, 1923			4,822,589
Burned			35,000
Stocks July 31, 1923, total			2,087,919
In consuming establishments, total		1,089,230	
In cotton-growing states	532,203		
In all other states	557,027		

In public storage and at compresses, total		938,689
In cotton-growing states	752,888	
In all other states	185,801	
Elsewhere (estimated)		60,000

(3) To determine "whether the bureau has sufficient authority to secure adequate information on the subject."

(4) To make recommendations for improving the methods of collecting and presenting cotton statistics by the Bureau of the Census.

Your committee has given consideration to these matters in the order thus indicated.

I. *Method*

The Bureau of the Census under authority of an act of Congress approved July 22, 1912, obtains from ginners, manufacturers and warehousemen information on the supply and distribution of cotton. It also obtains through the Bureau of Foreign and Domestic Commerce records of imports and exports. From these sources the statement of the supply and distribution is prepared. The various items under the heading of "Supply" are obtained through the agency of employees of the Bureau and statements by the ginners and warehousemen. On the other hand, the figures under "Distribution" are obtained through correspondence without the authority either to verify statistics furnished to employees of the Bureau or to make actual counts of the cotton in storage places. As a result, the opportunities for duplication and other error in the statistics of distribution are greater than in the statistics of supply.

It has been the custom of the Bureau of the Census at the close of each cotton year to present in a single statement, the figures of supply and of distribution. In view of the different methods employed in obtaining the two sets of figures it is not at all surprising that there should be marked discrepancies between them. Each year a difference has appeared and on a few occasions it has been significant. In these discrepancies we have found no evidence of any willful misstatement of figures by any of the informants or by the employees of the bureau. It has been the custom of the Bureau of the Census in making its annual statement to attempt, unfortunately, as we think, to balance the figures for supply and those for distribution by adding the requisite number of bales to the supply. The committee has found no instructions to this effect in any act of Congress and doubts the wisdom of continuing the practice. It is in this way that the disputed figure, 579,405 bales, in the report of August 18, 1923, had its origin.

II. *Accuracy*

Representatives of the committee have made as exhaustive a study of the data at the bureau and in the field as the time available would permit and have sought to verify these data through conferences with representative cotton statisticians, holders of cotton stocks, manufacturers and warehousemen. From these investigations it is impossible for the committee to state to what extent duplications entered into the report of last August, but after analyzing it the committee has agreed upon the following corrections which later it has entered in this report.

1. Cotton taken from the original bales as samples by buyers; cotton picked from bales damaged by weather or fire; and cotton from press sweepings, grouped together in the trade as "city crops," are estimated by the committee as at least 125,000 bales. While this would not increase the number of pounds it does increase the number of bales and should, therefore, appear on that side of the statement.

2. According to the annual report of the New Orleans Cotton Exchange supported by other data Mexican cotton to the amount of approximately 45,000 bales entered the country, without appearing as imports. These bales should also be added to the supply.

3. According to a statement from the Bureau of Foreign and Domestic Commerce supported by other data American cotton to the amount of 33,708 bales exported during the year was brought back into the country. This was reported in the exports but not in the imports and should, therefore, be deducted from the figures of distribution.

4. The report of the census was not supposed to include statistics of linter cotton, that is, the short fibre secured from re-ginning cotton seed. Evidence has been discovered, however, that 15,000 bales of such linters which had been bleached were exported as lint cotton. These 15,000 bales should likewise be deducted from the figures of distribution.

5. Investigation of certain storages showed duplications of mill and warehouse stocks to a degree which, if applied to other storages, would amount to not less than 25,000 bales. These should be deducted from the figures of distribution.

6. Foreign cotton to the amount of 20,171 bales which had appeared in the figures of imports was reëxported without appearing in the figures of exports. This cotton should be subtracted from the supply.

There is one other item of adjustment of which the committee was cognizant, namely, the ginnings in southern Texas prior to August 1, 1922 and 1923, but for various reasons thought it best not to include it. The committee has been convinced that the ginnings up to August 1, 1923, were in excess of those to the corresponding date in 1922, but because of the lack of data for 1922 it is impossible to definitely determine this adjustment. For this reason, the committee has preferred not to deal with this feature, especially as Congress has provided for an additional report of ginnings in the future prior to August 1, thus enabling the treatment of a statistical composite year in the future that will include ginnings for the twelve month period ending with July 31.

The effect of these additions to and subtractions from supply and subtractions from distribution is shown in the following revision of the figures of August, 1923:

REVISED STATEMENT OF SUPPLY AND DISTRIBUTION OF COTTON, EXCLUSIVE OF LINTERS, IN THE UNITED STATES FOR THE TWELVE MONTHS ENDING JULY 31, 1923.

SUPPLY	(*Bales*)
Stocks August 1, 1922, total	2,831,553
Ginnings, Crop of 1922	9,729,306
City Crop, rebaled samples, pickings, press sweepings, etc. (estimated)	125,000
Mexican cotton, not included in imports	45,000
Imports August 1, 1922 to July 31, 1923	469,954
Total	13,200,813
Less	
Reëxports of foreign cotton	20,171
Aggregate supply	13,180,642
DISTRIBUTION	
Consumption August 1, 1922 to July 31, 1923	6,664,710
Exported	4,822,589
Burned	35,000
Stocks at end of year, total	2,087,919
Total	13,610,218

Less		
Reimports of domestic cotton	33,708	
Correction for bleached linters exported	15,000	
Duplication in reports of mills and warehouses	25,000	
Total reduction		73,708
Total distribution less reduction		13,536,510
Excess of reported distribution over reported supply		355,868

The foregoing revised statement shows that the committee has been able to make material corrections in the August figures, but that a difference of 355,868 bales, about three fifths of the original difference of 579,504 bales, remains unexplained. It is impossible to determine just how much of this may be due to the underestimate of stocks and carry-over at the beginning of the year or to further duplication in mill and warehouse stocks. The important point is not to bring this discrepancy between the figures of supply and distribution into relief. The committee, therefore, recommends that in future statements of the Census Bureau no attempt be made to balance the figures of supply and distribution. The Census Bureau should issue figures which are the result of enumerations, avoiding estimates.

III. *Authority*

The committee agrees that regarding supply, the Bureau of the Census now has sufficient authority to secure adequate information. The committee does not believe that the bureau has sufficient authority for the collection of information on distribution. The present law does not give employees of the Bureau of the Census the right to examine books or other papers or to make an actual count of the cotton in storage places. The committee is of the opinion that such authority should be given.

IV. *Recommendations*

In addition to the above, the committee submits the following recommendations:

1. That an additional report to show the quantity of cotton ginned prior to August 1 be introduced as provided in a bill now pending.

2. That, in sections of the country where the entire crop has not been ginned by March 1, another ginning report be made for April 1 to ascertain the total amount of the crop.

3. That plans be developed to enumerate the cotton baled from samples and in pickeries, the so-called "city crops."

4. That the Department be urged to take action to secure an enumeration of the cotton now brought in from Mexico but not appearing in the imports statistics.

5. That the bureau plan to collect statistics of cotton consumption and of cotton held in storage at various points in such manner as to diminish the danger of overstatement or understatement of the supply. The committee believes that this end can be secured only by substituting collection through paid agents in place of the present method of collection by correspondence.

6. Reports on cotton statistics are now issued by two bureaus in the Department of Commerce, and by one in the Department of Agriculture. They are based in part on estimates and in part on enumerations, and the difference between them sometimes leads to serious confusion. These reports should, if possible, be coördinated under a committee or other harmonizing agency.

Respectfully submitted,

B. W. KILGORE, W. S. ROSSITER,
L. I. DUBLIN, W. F. WILLCOX,
Committee.

PROGRESS OF WORK IN THE CENSUS BUREAU

CENSUS OF INSTITUTIONAL CLASSES

The census of institutions or institutional classes, it may be recalled, covers hospitals for mental disease, institutions for feebleminded and epileptics, penal and reformatory institutions, almshouses, child-caring institutions and societies, other benevolent institutions, and general hospitals and dispensaries. The schedules for the various classes of institutions have practically all been received, and the material thus obtained is now going through the process of checking, editing, card punching, and machine tabulation. In the case of paupers in almshouses these processes have all been completed; and the card punching has been completed as regards institutions for the mentally diseased, feeble-minded and epileptics, and is under way for the other classes of institutions. Editing is still in progress on the schedules for jails and workhouses.

A series of state press summaries has been issued for each of the several classes of institutions giving number of inmates present and number admitted during the period covered by the census.

VITAL STATISTICS

The Manual of the International List of Causes of Death, embodying the revisions adopted at the Paris Conference of 1920, has now been published.

The Bureau of the Census is planning to print and distribute birth certificates for those registration states in which the officials having charge of the registration may wish to avail themselves of the coöperation of the Federal Government in this matter. The certificate will carry the facsimile of the signature of the director of the census and also that of the state registrar, through whom the certificates supplied by the Census Bureau will be issued.

The inclusion of Iowa in the birth registration area for 1924 brings the number of states composing that area up to 31. They comprise 74.5 per cent of the total population of the United States. The death registration area, to which Iowa was added in 1923, now includes 38 states and comprises 87.7 per cent of the total population.

BIENNIAL CENSUS OF MANUFACTURES

The bureau is beginning to publish in the form of press summaries the results of the 1923 census of manufactures, the first such summary to appear being that relating to steel pens which was issued April 9. This is about four months earlier than the appearance of the first press summary for the manufactures census of 1921. The improvement is partly the result of the fact that the bureau has succeeded in securing schedules by mail to a greater extent at this census than ever before, having secured reports in this way from more than one half of the total number of establishments as compared with about one fourth in the previous census.

CENSUS OF ELECTRICAL INDUSTRIES

The last quinquennial census of electrical industries covered the year 1922. The tabulation of the data has been completed and copy for the reports sent to

the printer. These reports will, as heretofore, be four in number, covering respectively electric railways, central electric light and power stations, telephones and telegraphs.

CENSUS OF WEALTH, DEBT AND TAXATION

The decennial investigation of wealth, debt and taxation is completed so far as the work of the Bureau of the Census is concerned. The Digest of State Laws relating to Taxation and Revenue, an octavo volume of 544 pages, has already been printed and distributed, and the copy for the remaining final reports which will cover respectively the subjects of (1) estimated national wealth, (2) public debt, (3) taxes collected, and (4) assessed valuation, has been sent to the printer.

J. A. H.

THE COLLECTION OF ORIGINAL STATISTICAL DATA

MEETING OF NEW YORK MEMBERS OF THE AMERICAN STATISTICAL ASSOCIATION

The spring meeting of the Association, which was held at the Aldine Club, 200 Fifth Avenue, New York City, on Friday evening March 21, 1924, taxed the capacity of the meeting room. The large attendance was attributed to the energetic and effective work of our Secretary in arousing the interest of the local membership in the subject of "Methods of Collecting Original Statistical Data." The speakers of the evening discussed the accuracy, completeness and representative character of the facts obtained through the different methods of statistical field investigation, and commented on the relative reliability of the several methods. This meeting disclosed the practical experience of the speakers of the evening with respect to the choice of samples and in the application of tests for representativeness and adequacy.

The meeting was conducted by Dr. Louis I. Dublin, President of the Association, who, in his introductory remarks, pointed out that in view of the increased number of specific statistical inquiries now being undertaken, and the immense amount of work necessary to cover any field adequately by the older methods of complete enumeration, the questionnaire, or random-sample, method would come into greater rather than into less use in the future. He urged that the membership of the Association direct their thought to the improvement of methods of collecting adequate and satisfactory data from representative samples, or from what he termed "limited universes."

The first speaker of the evening was Dr. Joseph A. Hill, Associate Director of the Bureau of the Census, who said that that office had made use both of the personal canvass and of the mail methods in collecting specific statistical information. He pointed out certain improvements in field practice which had resulted in enchancing coöperation between the industries of the country and the Census Bureau. He spoke particularly of the Bureau's effort to interest Chambers of Commerce and individual manufacturers in the completeness and accuracy of the census of manufactures.

Mr. R. S. Kellogg of the News Print Service Bureau complimented the Census Bureau upon its achievements in recent years in improving the scope and ade-

quacy of published results, and discussed the detailed methods by means of which trade associations secured prompt, accurate, complete and timely facts on the state of particular industries. He circulated some typical questionnaire schedules and emphasized certain points of excellence. He warned against the so-called "shot gun" questionnaire, and made a strong plea for simplicity, brevity and clearness in the questionnaires which are sent to manufacturers by governmental and private organizations.

Mr. Roswell F. Phelps, Director of the Division of Statistics, Massachusetts Department of Labor and Industries, gave an outline of the work of the old Massachusetts Bureau of Statistics and told how the organized Labor Statistics Service of the Commonwealth of Massachusetts was securing useful current data on manufacturing operations, building, employment and unemployment, and other pertinent facts on labor conditions in his state. He emphasized particularly the usefulness of a miscellaneous information service which would assist students of current labor legislation in the discussion of specific problems, such as the eight-hour day, the question of night work for women, etc. He gave specific information on the standards of representativeness and adequacy of the wage and employment data collected for Massachusetts industries by his department. He also exhibited certain graphical charts, and gave illustrations of the use of temporary trend data, pending the publication of comprehensive results for the whole state.

Mr. Paul T. Cherrington, Director of Research of the J. Walter Thompson Company, gave the members an interesting talk on the wide scope of the inquiries pursued by business research specialists into the facts of consumer demand and consumer opinion. He gave specific examples of the methods pursued by the statistical departments of advertising agencies in practical field work. He also reëmphasized the classic principles of questionnaire construction and pointed out the relative efficiency of the personal canvass and mail methods of securing information.

Miss Margaret L. Stecker of the National Industrial Conference Board presented a paper on the practical results obtained in her office with various types of questionnaires.

Mr. Ralph G. Hurlin of the Russell Sage Foundation discussed the data problems of governmental and private research organizations, and urged that further consideration be given to the matter of testing the adequacy of ratios and other derived values drawn from tabulated questionnaire returns.

Mr. E. B. Patton of the New York State Industrial Commission traced the history of the effort made by his office during the past twenty years in securing wage and employment data in the state of New York. He distinguished between the data which were susceptible to report and pointed out that at times it was necessary to use somewhat inferior sources of record, in the lack of better and more desirable original data. He also emphasized some of the classic principles of questionnaire practice, such as the pertinence of the inquiry, the direct relation of the schedule to the actual needs of the compiling office and to the needs of ultimate consumer of the results.

Mr. L. H. Haney, Director of the Bureau of Business Research, New York

University, outlined certain of the specific statistical inquiries pursued by his office in the city of New York.

The discussion was summarized by Mr. E. W. Kopf, who said that the purpose of the meeting was to secure an expression of experience of the membership with methods of collecting original data and to place the results of the meeting at the disposal of the committee which has in mind the preparation of the set of standard reference works proposed at the December, 1922, Annual Meeting of the Association. He emphasized the need for acquainting the newer members of the Association with the published works of the earlier American and British statisticians and urged particularly a study of the principles and practice of statistical inquiry as set forth by Francis A. Walker, Carroll D. Wright, and the other distinguished members of this Association who laid the foundations for the extensive statistical operations of the Federal and State Governments. He pointed out that the texts on statistics available to the present-day membership of the Association did not deal adequately with the distinguished achievements of the older school of statisticians in the United States and that much effort was wasted at the present time in learning all over again the lessons which were so clearly set forth by the fathers of this Association. In closing, Mr. Kopf suggested that the Committee on Educational and Professional Standards would be pleased to have an expression of opinion from the members at some future meeting on the subject of testing the sufficiency of questionnaire results, especially in respect to the dependability of conclusions derived from such tabulated materials. He held that this involved a thorough consideration of the calculus of probabilities and a rigorous discipline in quantitative logic. The work of the Committee on Educational and Professional Standards, he held, would be materially advanced if members of the Association would assist the committee in outlining courses of study in the cultural and technical elements of statistical work. He pointed out that the present membership of the Association constitutes practically a new generation of statistical workers and that the Association had both an opportunity and a responsibility for developing the professional competence of its members. He said that among the present membership of the Association there were perhaps many who were potentially capable of work which would measure up to the high standards set by the founders of the Association.

THE LEAGUE OF NATIONS HEALTH SERVICE

Approval of the work thus far accomplished and of the programme of activities for 1924 for the Service of Epidemiological Intelligence and Public Health Statistics were given by the League of Nations Health Committee at its meeting in Geneva in February.

The report of the Chief of the Service, Mr. Edgar Sydenstricker, was devoted largely to two main phases of the Service's work, namely, the international exchange of epidemiological information, and preliminary studies and activities looking toward the development of greater uniformity in vital statistics from the international point of view. This development of the epidemiological and statistical work of the Health Section has been made possible by generous grants from

the International Health Board, Rockefeller Foundation. The report shows that during the past twelve months the exchange of epidemiological intelligence was extended, so that at the present time current information on the prevalence of the more important communicable diseases is being received by the League of Nations Health Section from over seventy countries and colonies. The monthly publication (*Monthly Epidemiological Report*) of the Health Section was established in July, and in it are published current reports from the different countries for thirteen diseases. Quarterly supplements, containing information on other diseases, will be published and an annual summary for 1923 is now in preparation. A summary of the statistics of notifiable diseases in Europe, 1922, was published last October.

For the proper interpretation of the information currently received, as well as for the purpose of establishing a proper foundation for the study of international comparability on vital statistics, a series of statistical handbooks was begun. The volumes on Belgium and the Netherlands have already been published, and others are in preparation (for Great Britain, Spain and Portugal). It is planned ultimately to issue a handbook for every country, presenting a detailed description of the statistical procedure and practice now employed by each, and containing sufficient information to enable the vital statistician and epidemiologist to use and evaluate the statistics of that country. The immediate supervision of this work on the handbooks has been entrusted to Dr. Major Greenwood, Medical Statistician in the British Ministry of Health.

The aim of the Service has been to avoid the establishment of anything like a superstatistical bureau at Geneva; on the contrary, the policy has been, first, to get into close touch with the statisticians and statistical offices in the different countries; and secondly, to secure their collaboration in all the activities of the Service. For the European countries preliminary reports were prepared by the Ministries of Health and the statistical offices, at the suggestion of the Service, on the public health administration and statistical procedure. Two such reports, on the public health organization and administration of Germany and Austria, have already been published, and others are in the press.

Among the questions which it is proposed to take up are: The differences in the procedure for certification of causes of death and for classification of joint causes; the lack of uniformity in age groups in statistics of causes of death; the question of a standard population for the adjustment of mortality rates; and a uniform definition of still births. Relations with the International Institute of Statistics will be established, as well as with the statistical offices of the different countries, in the prosecution of this work.

It is expected that within the next few months committees will be appointed and that several statisticians will be invited to collaborate with the staff of the Service in the various phases of study of the questions involved. When any of these committees, after consulting the statisticians in the various countries, is ready to make a recommendation on uniform procedure on any point, its report will be placed before the Health Committee of the League for further action.

Owing to the fact that Mr. Sydenstricker's leave of absence of one year from the United States Public Health Service has expired, he is returning to his post

at Washington. This leave was extended by two months at the request of the League of Nations, but the request that he should be allowed to remain for a further period of two years was not found to be possible by the United States Public Health Service. Accordingly the services of Dr. Otto R. Eichel, Chief of the Division of Vital Statistics for New York State, have been secured for a period of six months, beginning March 1.

MISCELLANEOUS NOTES

CARL HOOKSTADT

Carl Hookstadt, who since September, 1915, had been employed by the United States Bureau of Labor Statistics, and who developed into the expert of that bureau on the subject of workmen's compensation legislation and administration, died in Saint Luke's Hospital, Saint Paul, Minnesota, on March 10. Mr. Hookstadt had recently been assigned a part in the drive the Bureau of Labor Statistics is making to secure better statistics on industrial accidents with a view to establishing a dependable accident rate by industries and departments within industries in the United States. He was very widely known, particularly among workmen's compensation men, who looked upon him as the final authority in all their problems. He was a comparatively young man, being only forty-four years of age, and his loss will be severely felt by the Bureau of Labor Statistics.

A New Index of Employment in Manufacturing Industries.—The United States Bureau of Labor Statistics has put out an index of employment in manufacturing industries based upon its enlarged series which was begun in July, 1922. As is quite generally known the bureau began in November, 1915, with thirteen industries and carried an index of these for a time, using January, 1916, as a base or 100 per cent. This index was carried up to December, 1921, but was discontinued for several reasons, the principal one being that the bureau had determined to enlarge both the volume and the scope of its employment reports. It now carries 52 industries instead of 13, and covers 8,222 establishments including 2,693,636 employees in these industries. An index based upon this number and using the monthly average for the year 1923 as a base or 100 per cent was published in the April, 1924, issue of the *Monthly Labor Review*. This index, however, went back only to July, 1922, the date of the enlargement of the scope of the returns. The May issue contains a general index and an index for each of the original industries upon the 1923 base, going back

GENERAL INDEX
(Monthly Average 1923 = 100)

	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924
January	...	105	117	115	110	116	77	87	98	95
February	...	108	117	115	104	115	83	88	100	97
March	...	110	117	116	104	117	84	84	102	..
April	...	109	115	115	104	117	84	83	102	..
May	...	110	115	114	107	117	85	85	102	..
June	...	110	115	113	109	118	85	87	102	..
July	...	111	114	115	111	110	85	87	100	..
August	...	110	113	115	110	110	86	88	100	..
September	...	111	111	114	112	107	87	91	100	..
October	...	113	113	112	107	103	89	93	99	..
November	104	115	116	113	110	97	89	94	99	..
December	106	115	117	114	113	91	90	97	97	..

to November, 1915. The bureau thus has an employment index upon a definite base adjusted monthly over a ten-year period and with the assurance that this is to be continued and its foundation broadened from time to time. This is a weighted index, the weights used being the number of employees in the various industries carried as shown by the census of manufactures for the year 1919. The several indexes by industries cannot be reproduced here for lack of space but the general index may be given as shown by the table on the preceding page.

ETHELBERT STEWART,
Commissioner of Labor Statistics.

Charlier Visits the United States.—Professor C. V. L. Charlier, Director of the Observatory at Lund University, Sweden, has been delivering a series of lectures at different universities throughout the United States. Arriving in New York the early part of March, he visited the Harvard Observatory and spent a few days with Professor Poor at Dartmouth. After delivering two lectures on "Statistics and Philosophy" at the University of Michigan, he visited the Yerkes Observatory in Chicago. The next few months of his stay will be spent at the University of California, where he has been invited to deliver the Hitchcock lectures and also to teach in the Inter-Session and Summer Session. Professor Charlier plans also to spend some time at Lick Observatory and at Mt. Wilson Observatory. He will then visit Japan and return via the United States.

Shifting of Occupations among Wage Earners.—Statistics on the shifting of occupations among wage earners have recently been studied by Louis I. Dublin and Robert J. Vane, Jr. Their conclusions are presented in a paper published in the April issue of the *Monthly Labor Review*. The basis of this article is an analysis of the death claims paid on industrial insurance policies for a period of three months, August, September and October, 1923. The cases analyzed were selected at random from among the claims so paid. A total of 4,198 cases are included and based upon these there is attempted for the first time to give a scientific answer to the question of the shifting of occupations in the lives of workers. The combined results show that only 41.5 per cent were found in the same occupation at death as at the issuance of the insurance. A higher proportion, 57.3 per cent remained in the same industry though not necessarily in the same occupation. The proportion remaining in the same occupation varied from a minimum of 23.7 per cent for saloon-keepers and bartenders to a maximum of 74.6 per cent for tailors. The indications are that the more skilled workers go later into other skilled trades, while the less skilled shift to common labor. Only 4 per cent of the painters, 7 per cent of the tailors and 13 per cent of the carpenters who changed their occupation are classified at death among common laborers, whereas 20 per cent of the iron-foundry workers, 25 per cent of the iron and steel mill workers, and 29 per cent of the janitors wound up at death in the common labor class; while 33 per cent of the painters, 20 per cent of the tailors and 15 per cent of the carpenters who shifted occupation went either into some other of the skilled building trades or into some other skilled trade. Commissioner of Labor Statistics Stewart, at whose suggestion this study was inaugurated, is planning to have a study made of the death claims for an entire year, which will be, it is believed, a sufficiently large sample.

The German Metal Workers' Federation.—The German Metal Workers' Federation, probably the largest federation of unions in a single industry to be found in the world, having in 1922 746 local unions connected with the federation, and a com-

bined total membership of 1,624,554, is described in an article by Fritz Kummer of Stuttgart, contributed to the April issue of the *Monthly Labor Review*. Prior to the war the only wage statistics published by the German Government were such as it could secure from the records of the trade unions. It is well known that the manufacturers would not furnish the German Department of Labor with information. The best and most comprehensive source of information to the government was the German Metal Workers' Federation. This organization maintained a most remarkable statistical bureau and by its influence was gradually inducing the other nationally organized trades to use the local machinery for gathering valuable statistical material. Mr. Kummer's article in the *Review* gives a comprehensive picture of the organization and its multiple functionings. The paper is translated by Mr. Alfred Maylander, of the United States Bureau of Labor Statistics.

Copenhagen Population Register.—The city of Copenhagen with about 57,000 inhabitants has lately made a complete register of its population, which will be kept up to date with due reference to births, deaths and migration. Each family has its card with all necessary particulars—place and date of birth, occupation and address—these cards being arranged according to streets and house numbers, whereas there is a duplicate series arranged alphabetically so that any person on the register can easily be found. There are practical arrangements with the adjoining municipalities, which together with the capital form "Greater Copenhagen" with upwards of 100,000 inhabitants. This register will be of great use in public elections and several administrative problems. It may also prove useful in statistical investigations even though an important link is still missing, a registry of births, deaths and marriages. It is hoped that all municipalities in Denmark will before long institute a register after the same pattern as that of Copenhagen.

Bank Debits to Individual Accounts.—An important addition to the available information regarding the trend of business in Canada will be made by the publication in future of a bulletin of Monthly Statistics of Bank Debits to individual accounts at the clearing house centers of Canada. The statistics of Canadian bank clearings, which have been in general use during the last quarter of a century, have become increasingly less indicative of the volume of business because of the many amalgamations which have taken place between banks. As a result of this amalgamation many transactions which were formerly between banks are now completed within a bank and no longer appear in bank clearings. The new record will also be more comprehensive than bank clearings in that it will include intra-bank as well as inter-bank transactions.

Galley Proof for Filing.—Requests have come to the editor that arrangements be made to furnish galley proof of the book reviews and announcements of statistical literature to such subscribers as may wish this material in a form more suitable for their filing systems. Such a service is furnished by other journals, as, for illustration, the *American Journal of Sociology*. The matter has been taken up with the Rumford Press, with the result that the announcement is here made that the galley proof of the book reviews and recent literature notices appearing in the JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION can be furnished at the rate of $1 a year if as many as 100 subscribers want it. Readers desiring this service should communicate with the editor or the secretary.

Employment Statistics in Oklahoma.—Oklahoma becomes the tenth state in which comparable employment statistics are being collected by or under the auspices of

state bureaus of labor statistics, the Oklahoma Department of Labor having begun the collection of monthly employment and earnings statistics for the leading manufacturing and mining industries of the state in January of this year. The Association's Committee on Governmental Labor Statistics was of assistance in developing the plan for these statistics. When a small appropriation for statistical purposes became available last year, Commissioner Claude E. Connally wrote to the committee asking if a representative could be sent to advise in planning for the collection of employment statistics. Dr. Don D. Lescohier of the University of Wisconsin represented the committee for this purpose.

Report of the Federal Reserve Board.—The annual report of the Federal Reserve Board for the year 1923 was presented to Congress on March 29, 1924. The report contains a full discussion of Federal Reserve policy and summary tables of the year's operations. A fuller report containing more detailed statistical data will be issued later. It is of interest to statisticians to note that the board devotes several pages of its discussion to the subject of guides to credit policy, among which an important rôle is ascribed to index numbers reflecting the volume of industrial and trade activity.

The wastage of men was the subject of an address delivered by Commissioner Ethelbert Stewart at the Chicago conference of the Association of Governmental Labor Officials of the United States and Canada. Commissioner Stewart dealt with the changing emphasis in the invention of machinery from the former effort to supplant the skilled labor to the present attempt to put labor saving devices in the place of the unskilled laborer. He stressed the value of this tendency as opposed to the old wasteful policy of unrestricted immigration and the resulting wastage of man power.

The operating costs of motor vehicles was the subject of a paper read before the Society of Automotive Engineers in New York, February, 1924, by Mr. R. E. Plimpton, Associate Editor of *Bus Transportation*. The paper dealt with a variation in costs over a ten-year period and was especially concerned with the detailed costs as affected by the age of the vehicle. An abstract of the paper with accompanying tables and charts was published in *Bus Transportation* for March, 1924.

John Biddle Garrett, who has been a member of the American Statistical Association since 1892, died at his home at Rosemont, Pennsylvania, at eighty-eight years of age. He was Vice-President of the Lehigh Valley Railroad, 1899, President of Girard Trust Company, manager of Haverford College, Trustee of Bryn Mawr College, and a Member of the American Academy of Political and Social Science.

Mr. Edgar Sydenstricker, who was granted fourteen months' leave of absence from the United States Public Health Service to serve as chief of the Service of Epidemiological Intelligence and Public Health Statistics of the League of Nations in Geneva, returned to Washington on March 24 to take up his duties again as statistician in charge of the Public Health Service's statistical investigations.

Professor Rainard Robbins, resigned his position on the Mathematical Faculty of the University of Michigan to become Assistant Actuary in the Department of Insurance of the State of New York. Professor Robbins is located in New York City at the Broadway office of the Insurance Department.

Dr. Otto R. Eichel has been granted a leave of absence by the New York State Health Department, of which he has been director for the last seven years, to direct the Epidemiological Intelligence and Statistics section of the League of Nations.

ACTIVE COMMITTEES, 1924

1. *Committee on Governmental Labor Statistics:*
   Mary Van Kleeck (Chairman), A. J. Altmeyer, Charles E. Baldwin, Joseph A. Becker, W. A. Berridge, W. Randolph Burgess, R. D. Cahn, Frederick E. Croxton, J. Frederic Dewhurst, Don D. Lescohier, M. O. Lorenz, Royal Meeker, Eugene B. Patton, Roswell F. Phelps, Walter W. Stewart (alternate from the Federal Reserve Board), Woodlief Thomas, Leo Wolman, F. G. Tryon, Ralph G. Hurlin (Secretary).
2. *Committee on Institutional Statistics:*
   Horatio M. Pollock (Chairman), Edith Abbott, Kate H. Claghorn, Edith M. Furbush, Joseph A. Hill, Robert M. Woodbury.
3. *Joint Advisory Committee on the Census:*
   W. S. Rossiter (Chairman), Robert E. Chaddock, Carroll W. Doten.
4. *Committee on Educational, Scientific and Professional Standards:*
   James W. Glover (Chairman), Leonard P. Ayres, W. A. Berridge, R. E. Chaddock, Edwin W. Kopf, Edmond E. Lincoln, Malcolm C. Rorty.
5. *Committee on Fellows:*
   Leonard P. Ayres (Chairman), term expires 1924; Warren M. Persons, term expires 1925 (appointed to fill the unexpired term of John Koren, deceased); Edmund E. Day, term expires 1926; Joseph A. Hill, term expires 1927; Raymond Pearl, term expires 1928 (appointed to succeed C. W. Doten for five-year term).
6. *Committee to Coöperate on the Project of an Encyclopedia of the Social Sciences:*
   Walter F. Willcox (Chairman), Warren M. Persons, Mary Van Kleeck.
7. Appointment of Malcolm C. Rorty January 10, 1924, term of three years, to represent the Association on the Board of Directors of the National Bureau of Economic Research.
8. *Committee on Classification of Technical Statistical Workers in Government Service:*
   Edwin F. Gay (Chairman), Kate H. Claghorn, Malcolm C. Rorty.
9. Representatives of the American Statistical Association on the Social Science Research Council, appointed by Warren M. Persons, November 30, 1923: Walter F. Willcox, three-year term; Edmund E. Day, two-year term; H. L. Rietz, one-year term.
10. *Committee on Finance:*
    Carl Snyder (Chairman), H. A. E. Chandler, Robert E. Chaddock.
11. *Committee on Membership:*
    Edwin W. Kopf (Chairman). Power granted to the Chairman and to the Secretary of the Association to add such other members as are deemed desirable.

MEMBERS ELECTED SINCE FEBRUARY

Alexander, Magnus W., Managing Director, National Industrial Conference Board, New York City.
Bates, Maria W., National Organization for Public Health Nursing, New York City.
Baxter, William J., President Clothing Forecaster Corporation, Worcester, Mass.
Boardman, K. S., Bureau of Business Research, Cambridge, Mass.
Buechel, F. A., Agricultural Administration, Agricultural College of Texas, College Station, Texas.
Chen, Ping C., Banking, Hongkong, China.
Clement, G. Y., American Educational Association, Philadelphia, Pa.
Connor, J. N., American Radiator Company, New York City.
Dalrymple, A. G., Canada Life Assurance Company, Toronto, Canada.
Davis, Jerome, Yale University, New Haven, Conn.
Dockendorf, John, Public Service Building, Milwaukee, Wis.
Eaton, Thaxter, Harvard Economic Service, Boston, Mass.
Feiss, Richard A., General Manager, The Joseph Feiss Co., Cleveland, Ohio.
Fernandy, Francisco M., Former Health Officer Cuban Government, Professor, Havana University, Havana, Cuba.
Fox, Martin J., A. S. Rosenthal Company, New York City.
Hall, Warren S., LaSalle Extension University, Chicago, Ill.
Hexter, Maurice B., Federated Jewish Charities, Boston, Mass.
Johnson, Lester D., United States Tariff Commission, Washington, D. C.
Kawashima, Ryoichi, student at Columbia University, New York City.
Killough, Hugh B., Graduate Student, Columbia University, New York City.
Koenig, Paul L., State Department of Agriculture, Harrisburg, Pa.
Lomholt, Einar, Western Electric Company, New York City.
McNair, M. P., Bureau of Business Research, Cambridge, Mass.
Mabey, O. Louis, Western Electric Company, New York City.
Maciaszek, John A., student, Northwestern Business School, Chicago, Ill.
Mertzke, A. J., University of Wisconsin, Madison, Wis.
Palmer, G. T., American Child Health Association, New York City.
Peabody, Leroy E., National Transportation Institute, Washington, D. C.
Pooley, Enid M., Institute of Social and Religious Research, New York City.
Schaffer, John G., University of California, Berkeley, Calif.
Smit, Andries A., student, Cornell University, Ithaca, N. Y.
Smith, Edgar L., Low, Dixon & Co., New York City.
Steeves, Maud E., Metropolitan Life Insurance Company, New York City.
Stryker, Roy E., graduate student, Columbia University, New York City.
Watson, Robert B., National Tuberculosis Association, New York City.
Weiss, Harry B., State Department of Agriculture, Trenton, N. J.
Wells, James E., Jr., Federal Reserve Bank, San Francisco, Calif.
Williams, John H., Consulting Management Engineer, New York City.
Winslow, R. M., Industrial Engineer, Minneapolis, Minn.
Wood, E. R., Kansas State Teachers College, Emporia, Kansas.
Young, Frederick P., Western Electric Company, New York City.

CORPORATE MEMBERS

Prudential Insurance Company of America, Newark, N. J.
Western Electric Company, Inc., 195 Broadway, New York City.

REVIEWS

Germany's Capacity to Pay: A Study of the Reparation Problem, by Harold G. Moulton and Constantine E. McGuire. New York: McGraw-Hill Book Company. 1923. xiii, 384 pp.

With this volume the publications of the Institute of Economics make an auspicious beginning. No more important problem could have been discussed. The treatment is thorough and painstaking and the general influence of the book should be wholesome. If the book errs, as I believe that at some points it does, it is merely because it pushes a good case a little too far.

The first chapter, more than any other, suffers from this lack of moderation. It expounds the perfectly valid theorem that a country's capacity to pay foreign debts is measured by the amount by which its exports can be made to exceed its imports. But the practical conclusions it draws from that theorem are too rigid and uncompromising. For example, it is going too far to characterize as "absurd" the notion that "Germany can pay the European Allies by borrowing from the United States or in neutral countries," or to wave aside as mythical the notion that the transfer to the Allies of equities in property *within* Germany might be counted as payment on account of reparations.

I do not mean to suggest that such methods offer a solution of the reparations problem, or that it would be wise or practicable to stipulate that they should be made part of any formal scheme or schedule of payments. I mean merely that back of these notions, so commonly disparaged by economists, are two fundamental truths which economists cannot afford to neglect.

In the first place, there is an important difference between a *legal* payment and an *economic* payment. It is the former upon which the reparations problem turns. Messrs. Moulton and McGuire concern themselves only with the latter. A legal obligation may be discharged by refunding a debt,—even though new obligations are created. If, in the long run, the possibility of refunding the debt made no difference in the volume of the actual (economic) payments that have to be made in a given period of years, the distinction I am emphasizing would have little significance. But in fact—and this is the second point—it really makes a substantial difference. Who shall say just when the economic (as distinguished from the legal) payment of the indemnity of 1871 was completed? And just what does the growth of British foreign investments during the nineteenth century suggest with respect to the problem of final or economic payment? Is it at all certain that, up to 1914, we had, in any real sense, ever completed the economic repayment of our earliest borrowings from England?

The creation of any large foreign debt is bound not only to affect the immediate currents of world trade, but also to modify, for an indefinite period, the world's balances of international indebtedness. If Germany's obligations were set at a reasonable sum, and if, at the end of the stated term of years, the legal payments were finally completed, it would inevitably be found that some considerable part

of those obligations had been refunded in a thousand and one different ways, and just as certainly France's net outstanding international credits would be found to be notably larger than otherwise they would have been. In that way, rather than in goods, she would have collected part of the payments due her.

Of course there is nothing in these considerations to justify the figures at which Germany's debt has been fixed by her creditors. To support refunding operations of any sort Germany would have to prove her ability to pay a little more than interest upon the amount of the debt. Thus far most of the negotiations with respect to reparations have run in terms of amounts upon which—for many years to come—Germany could not even pay interest. Moreover, the study of her possible export surplus affords the only practical basis for a reasoned judgment respecting Germany's capacity to pay. My objection to this first chapter is merely that it over-simplifies an exceedingly difficult and complex subject and that its tone is needlessly and even irritatingly didactic.

In the second chapter, dealing with Germany's international balance of accounts, the discussion reaches firmer ground. The analysis of the balance of commodity trade, in particular, is exceedingly well done. The problem of the probable amount of Germany's foreign investments is carefully discussed, and the amount is put at 20 billions of gold marks in 1914 and 2 or 3 billions in 1923. The second figure is reached by a process of subtraction and therefore hinges upon the first. The first, despite the fact that it is reached after a survey of nearly all the more important estimates that have been made, and takes account of the German census of foreign-owned securities in 1916, I believe to be too low. A list of German-owned property in Latin America, compiled by the Bureau of Enemy Trade of the War Trade Board, indicated that the total for that region alone amounted to from 4 to 8 billion marks. The German-owned property liquidated or sequestrated in other countries was worth more than 10 billions. In Latin America, as elsewhere, a large part of these holdings were not of a sort that would have been disclosed by the German government census of 1916. For these reasons, and for others which I cannot recount here, I conclude that the total value of German foreign holdings in 1914 was probably not less than 30 billion gold marks. The net shrinkage during the ensuing eight years may have been as much as 20 billions. Of the 10 billions that may have remained I doubt that more than 4 or 5 were liquidatable, in the sense that they might conceivably be converted into funds that might be transferred to Germany's creditors.[1]

By all odds the most valuable chapter in the book is the third, on "What Germany has paid and how." By carefully reviewing and analyzing the complicated records that bear upon the case, the authors have rendered an important public service. Germany has claimed that up to October, 1922, her payments had aggregated 45 billion gold marks. The Reparation Commission, however, up to the end of 1922, had credited her with only 8 billions. The authors' esti-

[1] This review was in type before the McKenna committtee made its report. Its estimate of Germany's prewar foreign holdings is in substantial agreement with the one given above. For 1923, however, the committee's estimate is 6 billions (not less than 5.7 and not more than 7.8 billions)—a figure about midway between the Moulton and McGuire estimate and my own. The committee had fuller sources of information than were available to me, and I do not doubt that its estimate is nearer the truth than mine.

mate of the *cost to Germany* up to October, 1922, is 26 billions. Part of the difference between the German figures and those of the Commission is attributable to the inclusion in the former of certain items which the Commission had not yet entered as credits (such as allowances for some of the State property in ceded territories), and of other items (such as Germany's claims against her allies in the war, and the German-owned property seized or liquidated abroad) which cannot now properly be credited as payments on account of reparations. But differences in valuations are also responsible for a considerable part of the discrepancy.

It is going somewhat too far to say that the Commission has "uniformly taken, as the value of a commodity, the . . . price at which it could be liquidated at a forced sale." Nevertheless the credits that have been allowed Germany are in many cases unduly and in a few cases (notably in the value given the shipping transferred by Germany) iniquitously small. On any ground, economic or political, these low valuations are indefensible. Mutual jealousies among Germany's creditors, rather than any concerted policy, are probably responsible for them. But that does not help matters. Nor does the fact that many of the German claims are absurdly large. There is a difference between an advocate's brief and a judicial decision.

Nevertheless, some comment I have seen has exaggerated the significance of the difference between the Commission's credits and the figures reached by Messrs. Moulton and McGuire. In the first place, these figures, I infer, are not independent valuations, reached by independent research, but are rather more like arbitral awards as between the German claims and Commission's credits. Thus the railroad rolling stock delivered under the Armistice agreement is set down (as a *cost* to Germany) at a figure "fairly close to the German total." Scrapped military material is put at the German figure rather than at "the bare and unexplained figure" given by the Commission. The estimate of 650 million gold marks for the value of the Saar mines appears to be a mean between the German claim of 1016 millions and the Commission's allowance of 300 millions. An estimate made in 1919 by Dr. C. K. Leith and checked by other American mining experts, was only a trifle higher than the amount credited by the Commission. In general the findings of Messrs. Moulton and McGuire respecting these matters appear to give too much weight to the exaggerated German claims.

In the second place, the authors include several items, such as German property liquidated or sequestrated in other countries, which cannot enter at present into the reparations account. On a comparable basis, their figures, instead of being more than three times as large as those of the Reparation Commission, are little more than once and a half times as large.

In the third place, the purpose of the authors is not to estimate how much Germany has *paid* on reparations account, but rather to determine "the tangible economic value of what Germany has lost, in so far as the items capable of credit on reparation account are concerned." Their figures purport to express neither the economic gain to Germany's creditors nor the real economic burden imposed upon Germany, but their value within or to Germany as part of a going concern. This is a perfectly feasible object of inquiry, but its precise significance for the particular problem in hand is not clear. Combined with estimates of the values

actually received by Germany's creditors, it would have thrown light upon one of the major difficulties of the reparations settlement, namely the enormous shrinkage in value which attends the transfer of some kinds of assets. But such estimates are not given. As they stand, the figures are likely to be misinterpreted.

The fourth chapter deals with Germany's foreign trade requirements. It is concerned, for the most part, with Germany's minimum import needs and her possible export markets. Statistics carry one only a certain distance into such problems. Beyond that point they are matters of judgment. It is possible—at the expense of a good deal of pains—to adjust the statistics of Germany's pre-war production, consumption, and foreign trade, so as to take account, quite accurately, of her territorial losses. But the results would be misleadingly precise, and I believe that such rough adjustments as are made by Messrs. Moulton and McGuire (on the basis, for example, of the loss of population) are better.

There is much careful analysis in this chapter, but the conclusion, that "German exports must equal 14 billion gold marks merely for the purpose of paying for the necessary imports," is marred by a serious error. The imports of raw materials and manufactured goods required are estimated upon the assumption that German factories, whether producing for the domestic or the foreign market, are to operate at something like pre-war capacity, that is, that exports are at their pre-war volume. Properly stated, the conclusion is that if Germany's exports should equal (roughly) 16 billion gold marks, her imports could not be less than 14 billions. (This conclusion, I think it proper to say, is in fairly close agreement with some unpublished estimates of my own.) It would have been better, of course, to have distinguished between Germany's minimum import requirements for domestic needs and the variable import requirements of her export industries. Figured in this way the outlook for a possible favorable balance of commodity trade is not quite so hopeless as the authors make it appear.

In the fifth chapter, which discusses the German budget, there is much material which will be new to American readers. This chapter embodies the fruits of independent research and is an important contribution to our knowledge of the subject. The authors' own summary of it is as follows:

> (1) During the war, for reasons of political expediency, taxes were kept relatively low. (2) In the fiscal years 1919–20 and 1920–21, demobilization, internal reconstruction, and treaty fulfillment required enormous expenditures to be made *at once*, while the necessary reorganization of the fiscal system delayed the collection of taxes. (3) In 1921–22, the tax program was more effective, perhaps as much as one fourth of the national income being taken by the government. In spite of this, the budget was still far from balanced, for the increased outlays under the terms of the London ultimatum quite off-set any gains otherwise made. (4) In 1922–23, the almost uninterrupted fall of the currency completely demoralized the fiscal system.

The finding that in 1921–22 taxes took perhaps as much as a fourth of the national income hinges upon very uncertain estimates of the aggregate amount of that income. The authors' own estimate is made up of three elements: (1) Helfferich's estimate of 40 billion gold marks for 1913; (2) an estimate that the volume of physical production in 1921 was 75 per cent of what it was in 1913;

(3) an estimate, reached by a curiously roundabout method, that prices and incomes in 1921 were 28.5 times as high (in marks) as in 1913. In my opinion these intermediate estimates are reasonable, though highly conjectural. The conclusion reached, while necessarily subject to a large margin of error, is probably not so far from the truth as to be wholly misleading.

There is less that is new in Chapter VI, on the German monetary situation. The purpose of the chapter is mainly expository. Few economists will quarrel with the conclusion that paper money inflation in Germany "was not the result of a deliberate policy designed to bankrupt the country financially and thus to make reparation payments impossible." In fact, I think the authors hardly attach sufficient importance to the leading rôle played in the progress of German inflation by the unremitting pressure for foreign payments. In their view inflation was the necessary result of an unbalanced budget. They fail to note that foreign debts are even more potent causes of inflation than are domestic debts, and that the pressure of foreign debts, more than anything else, made the balancing of the German budget impossible.

After a short but informing account of how France met the indemnity of 1871, the authors come, in their last chapter, to a general summary of the bearing of the general European economic situation upon their problem. Here they emphasize the malevolent effects of "far-reaching maladjustments in the larger economic organization," and of "the disruption of the international financial and economic system." "Germany will regain economic health and strength only as the rest of Europe recovers from the disastrous shattering of established economic relationships that has resulted from the war." All of which is undoubtedly both true and important. And yet such experience as the world has had with these matters indicates that, given a wholesome financial situation, economic recuperation is likely to be achieved rapidly. The loss of markets is an affair of relatively small account as compared with the graver matters the authors have passed under review in their earlier chapters.

If I have overemphasized the particular points at which the authors' conclusions seem to be questionable or at which they seem to overstrain their argument, it is because I have assumed that their theses and their conclusions are, in general, such as must commend themselves to all economists. Certainly, by their painstaking examination of the problem they have put all economists in their debt.

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The Elements of Vital Statistics in Their Bearing on Social and Public Health Problems, by Sir Arthur Newsholme. New edition—entirely rewritten. London: Allen & Unwin, 1923. 623 pp.

The preceding edition of this work appeared nearly a quarter of a century ago and has long been out of print. For many years it had been the best introduction available in English to vital statistics if not to statistics *sans phrase* and merely a new edition would have been welcome. But this is more than a new edition; it is a new work and larger than the edition of 1899 by more than one half.

In England since then the organization of public health has greatly improved and in the United States the national movement has been created. The preceding edition appeared as the physicians of the American army were beginning to transform Cuba and shortly before they transformed the Panama Canal Zone from pest holes to marvels of healthfulness. At about the same date the Bureau of the Census became a permanent office and was enabled to coöperate as never before with American states and cities in laying the statistical foundation of that public health work to the possibilities of which the experiences in Cuba and Panama had so dramatically called American attention. No American book could then have been put beside Newsholme's Vital Statistics, even though for American purposes it was confined somewhat too closely to topics which especially interested English sanitary and medical statists. In the interim several American books and many articles have appeared in its field some of which might have rivalled a new edition of the book. The progress of the American public health movement since 1899 is indicated by the fact mentioned in the preface that the most urgent requests for a new edition came from this country. The author has complied not by a new edition and not merely by a new work, but by one prepared to satisfy these American demands, a task for which his visits to the United States and his residence and lectures here admirably equipped him. Most of his illustrative material is drawn from the statistical reports of England and the United States with only an occasional reference to those of Scotland, Ireland or the British dominions. Indeed the contents of the book would have been indicated more precisely by its title had it been called Vital Statistics of England and the United States.

Dr. Newsholme is not of the school of mathematical statists; his work "is intended for those not equipped for higher mathematical investigations" (page 6); he insists that correlations which are not *a priori* probable but are supported by mathematical evidence are to be accepted only with great caution (page 550); and that a high coefficient of correlation between two factors may give a misleading impression of the relation between them (page 521).

Dr. Newsholme, believing in the powerful influence of environment, is skeptical of results which run counter to that view. Thus he finds no convincing evidence that a high mortality among infants and children is attended and in a measure counterbalanced by a low mortality at greater ages because the average health of the sifted survivors is better (page 359); he believes that infection exerts a major influence upon the dissemination of tuberculosis and that the inheritance either of the disease or of an impaired power of resistance is a minor factor (page 543); he challenges the evidence offered to show that drunkenness in parents has no influence upon the physique or intelligence of their children (page 547).

New chapters treat of the statistical aspects of the following subjects: poverty and hospital provision, housing, alcoholism, pneumonia, venereal diseases. Even more interesting and important than these is Part II, "Methods and Miscellanea," containing nine chapters mainly new on the study of causation, correlation, tests of physical and mental efficiency, graphic methods and fallacies. The work is to be welcomed as doubtless the best handbook in English for the public health worker, American or English, and a very good, perhaps the best,

general introduction to statistics. Its great strength lies in the author's intimate first-hand knowledge of public health work at its best and his sound judgment about the information needed by public health workers. In the new work a long step has been taken towards the ideal treatise on vital statistics.

But after according it this cordial welcome some questions may be raised for the consideration of our successors. First, what is vital statistics? It is, he tells us, "the science of numbers applied to the life-history of communities and nations" (page 19). This definition leads one to infer that Dr. Newsholme would define statistics as the science of numbers, the adjective in his title merely limiting its applications. Statistics has hitherto been defined in scores of ways, but not, so far as I know, as the science of numbers, and that definition seems to me unsatisfactory. We ordinarily think of mathematics in its various divisions, or many of them, as the science of numbers, and surely statistics is not a branch of mathematics. The definition assumes, too, that statistics is a science and not merely a method; in that assumption it runs counter to the prevailing English view. The study of vital statistics as a well-defined subject is almost peculiar to the English-speaking world. This is indicated by the fact that none of the other great languages has a word or phrase exactly synonymous with vital statistics in the sense in which Dr. Newsholme employs it. The nearest equivalent, as he recognizes (page 20), is demography or theory of population, but that word covers a broader field than Dr. Newsholme's Vital Statistics. It seems better frankly to make vital statistics synonymous with demography and to define it as the study of human population by the aid of the statistical method. This definition avoids the contentious question whether statistics is a science, the answer to which depends mainly upon the definition of science; it avoids, also, an emphasis upon history which wrongly narrows the field. A census, being a record of the condition of a population at a given moment, has little direct bearing upon history and yet every discussion of vital statistics should begin, as this does, with a statement of census results.

Another question raised in my mind by the book is this: Should vital statistics be treated as a national or as an international subject? It has been developing for nearly a century and, no doubt, England has contributed to it more than any other country. But several other European countries have made substantial additions and an ideal manual of vital statistics should draw the best from them all. The three books that by the point of view of their writers and by the topics treated seem to come nearest to this of Dr. Newsholme are perhaps von Mayr's Bevölkerungslehre (1897), Westergaard's Lehre von die Mortalität und Morbilität (2d. ed. 1901), Prinzing's Handbuch der medizinischen Statistik (1906). The two books which have succeeded best in doing for continental countries what Dr. Newsholme did in his earlier editions for England and has done in this for England and the United States are perhaps Levasseur's La Population Française (1899-92) and Rauchberg's Die Bevölkerung Oesterreichs (1895). To none of these works have I observed a reference, and omissions of this sort reveal some weakness in the author's equipment on the historical and comparative sides, the obverse of his strength in the English and American fields. This defect weakens his discussion of certain minor topics. For example, he accepts M. March's con-

clusion that the death rate of married men is much below that of single men or widowers but hesitates to infer that this shows the influence of married life on vitality (page 214). It would have strengthened his discussion if he had referred to Westergaard's evidence that the deaths of Episcopal and nonconformist clergy in England are about three fourths of the expected number while the deaths of the Catholic celibate clergy are more numerous than would be expected, or to Kiaer's evidence that in Sweden the death rate of all males instead of increasing with advancing years actually falls after the age of 24 is reached and at 34 is about seven per cent below what it was ten years earlier in life, a unique deviation from the general rule of an increasing death rate with advancing years through adult life, which Kiaer explained by the steady transfer through those years of men from the less healthy state of bachelorhood to the more healthy state of marriage.

WALTER F. WILLCOX

Mankind at the Crossroads, by Edward M. East. New York: Charles Scribner's Sons. 1923.

Professor East's *Mankind at the Crossroads* is a well-written and thought-provoking book on the population question. The author is concerned with the increase in the human stock during the last century and forecasts the events of the next few hundred years. He takes serious note of the food supply necessary to maintain an increased population, not only in comfort but with an ever-rising standard of living. He has rendered a commendable service in crystallizing this problem. Nothing can be more important to us than to know whither we are going, how we are increasing, whether consistently with the best eugenic principles or whether indifferently and without provision for the future. It is vital that we know whether we shall be prepared to meet the needs of the future as to food and other necessities which make life agreeable or otherwise. To clear up uncertainty on such questions is the great function of science in its service to mankind. The conclusions of science will add to our control and replace blind drifting with definite constructive policy.

Professor East is a biologist of distinction. His essay is in the form of a biological excursion into the present relation of the human population to its environment. But, almost from the very beginning, the author confesses to a peculiar, indeed to an obsessing bias. While he is apparently very much concerned with the facts both of population increase and of possible food production in the future, he admits a conviction even before he has exhibited his facts. This conviction is that mankind is already dangerously near serious trouble, because of its encroachment on the limit of the food supply. As to the future, the situation must become quickly worse. Civilization is doomed unless the present increase in the population is materially curtailed. There is only one way of escape, namely, through the worldwide practice of birth control. The author uses his entire opportunity to appeal for birth control as a general practice. It undoubtedly mars the unquestioned merit of this book that the author does not allow the accumulation of fact to lead inevitably to his conclusion. The very insistence of his remedy from the very beginning puts the reader on his guard against what seems like special pleading.

It would be entirely unfair to suppose that the author has not marshalled a formidable argument for his thesis. He points out that in the last century the world population has been constantly increasing and has, in fact, more than doubled in this short period. It has taken half a million years for the human family to reach a total of eight hundred and fifty millions and only one hundred years more to reach a population of one billion seven hundred millions. Mankind is increasing at the present time at an even greater rate, which he estimates to be about twelve millions annually, or enough to double the population in about sixty years. In a little over a century at the present rate of going, the world population would amount to five billions two hundred millions, the maximum according to the author which could possibly be supported on the land area of the globe under present conditions of food production. Long before that time, pressure on our food supply would bring down standards of living and bring about war between nations. The whole social structure would be threatened. To permit such increase over a longer period would aggravate these disorders and would be both stupid and criminal. "The world would be filled with people without faith or hope, a seething mass of discontented humanity struggling for mere existence within the span of a single lifetime."

These serious and even ominous conclusions would follow if two assumptions of Professor East's were correct: first, that the population is now actually increasing at the rate supposed, and that such a rate can and will be continued for long periods, and, second, that the total food supply available on this earth could support only a population three times the present one.

The author's first contention is far from proved. While the world's population has markedly increased during the last century, there are already striking indications that this condition is righting itself. Never before in registration history has the birth rate declined as rapidly as during the last five years. The decline, moreover, has every earmark of permanence. It is possibly the result of the wide-spread propaganda for birth control, coupled with the unfavorable economic condition of many countries throughout the world since the war. Professor East is not unaware that in 1922 the birth rate in the United States was not in excess of 23 per 1,000; in England and Wales it was 20.4 and in Germany, 23.6 per 1,000. These figures are about a fifth below the rates of only fifteen years ago. The birth rate is now falling faster than the death rate, much as the latter is improving. The natural increase of population in the United States is no longer in excess of one per 1,000 annually. Even this increase will not continue for long if the marked changes that have occurred in recent years in the country's immigration policy become fixed. A considerable part of the country's natural increase in population is accounted for by the fertility of the new immigrants coming into the country as young and vigorous adults greatly improved economically in their new environment. It is they who are still breeding freely. This is obviously the picture in New York, Chicago, Boston, and the other large centers of the country, where a goodly part of the population are first generation Italians, Jews, Russians, and other Slavs. They are maintaining for a while their high birth rates but the restrictions on their continued coming will soon put a stop to the increase from this source. The second generation of these new-

comers has already shown a capacity to control their birth rate almost to the point of their American neighbors. We can see the birth rates of our industrial cities declining further and even rapidly in the course of the next generation. It is, therefore, not at all clear that the excess of births over deaths will continue to be anything like one per 1,000 per year. Especially is this true if we consider further that, with the reduction in the birth rate, there must follow ultimately an increase in the death rate because of the rise in the average age of the living. What is happening here because of changed habits of life and the modification of our immigration policy is happening among other white races because of the economic pressure and the unsettled political conditions following the war. In a number of large German cities, the death rate exceeded the birth rate last year and this condition is now fairly common in central Europe. There is no good ground on which we can assume that the rapid increase in population which prevailed before the war will continue undiminished during the next fifty or one hundred years. Professor East's book which appeared in 1923 unfortunately utilized the birth and death figures of five years ago. Reference to the vital statistics of the more recent years would have tempered his pessimism.

The author's second thesis, namely, that the earth can support decently only a population of about five billion persons is also very seriously subject to question. The discussion, Chapter IV, is the most instructive of the book and is well within the expert field of the author; but it is nevertheless true that he takes for granted much that is yet very unsettled and controversial in the field of agricultural economics. Thus, it is entirely too much to assume as he does that such vast areas as Australia, Africa and South America can produce only very limited additions to the food supply of the future—in fact, that these countries will cease to be exporters of food in from thirty to fifty years. The author's assumption that only the outer rim of Australia is capable of agricultural exploitation in the future is seriously undermined if a news item of the other day turns out to be true. An explorer appears to have covered the continent from one corner to the other and to have found the interior open to productive agriculture, provided care were taken to irrigate this area. Far from being a hopeless desert, Australia, according to this explorer, is a promising field for future food supplies. In the same way, there is good reason to consider Equitorial Africa, Siberia and some of the other arid regions which Professor East discounts altogether, as potential food centers of no mean proportions which can be tapped by resourceful man as the occasion arises. Even Europe itself is far from having exhausted all of its possibilities for self-support. European Russia is by best accounts only 55 per cent worked. To this possible increase of arable area should be added the greater returns all the world over which improved methods of agriculture will undoubtedly bring about.

Perhaps the most serious fault to be found with Professor East's discussion of this subject is his utter disregard of the effect of inventions and of the advancement of science on food production. No word is said of the possibilities for agricultural development from the production of cheap fertilizer on a large scale through the harnessing of the world's water power and of the other sources of giant power. States like Pennsylvania and Ontario, in Canada, are showing

what it means to the farmer to cheapen the cost of electrical energy. The possibilities are enormous and the surface has been scarcely scratched. When one considers the progress made in agriculture in the last half century through the application of machinery and the spread of education among the farmers, it is inconceivable that any discussion of our ability to take care of ourselves in the future should disregard this item of progress altogether. Certainly, agriculture is still one of the most backward of our activities and large resources will undoubtedly be tapped when agriculture comes into its own.

My feeling in closing this book is that the author has indulged his pessimism too freely. Birth control is not a panacea to be prayed for as a solution of all our ills. It is with us now and on a huge scale. Too few realize the extent to which the propaganda has permeated our social structure. It is a pity that this practice should go unchallenged receiving sanctification at the hands of serious students of the population question. Its effect on population and social welfare is already far from the blessing that its proponents consider it to be. The birth control movement has had its greatest effect on those groups of the population whom it is for obvious reasons a great misfortune to have less well represented in the populations of the future. The least effect has been felt among the illiterate, the poor, and the wretched, among whom a curtailment of reproduction would be a social blessing. They continue, as the author points out, to breed on without much restraint. It is the great body of the middle class and of the select who under the influence of this new teaching and because of economic pressure find it easier to restrict their fertility to socially dangerous limits. That is not how a strong and vigorous population is created. It is, in fact, a good way to bring about decadence. Professor East knows that the cream of the American population which he wants represented to a large degree in the next generation is not now reproducing itself. He knows the history of the men and women of education; they will be replaced from below, not from their own ranks in the next generation. How many families are there in the faculties of the Universities of America and in the families represented by the student body where the number of children born of a completed marriage is four? I venture that the proportion of such families is very small. Yet, it will take just about four children on the average to replace the two parents.

It would, therefore, appear that the author's fears are largely misplaced. There is not as great a danger of overpopulation as he fears. There is, however, very serious danger from the inconsiderate dissemination of birth control propaganda which in the nature of the case is almost limited in its operation to the best people. There would be no complaint if such propaganda could and did succeed in the lowest third or fifth of our population. But, experience has shown that it is almost hopeless to expect relief through this means. I am agreed with the author when he counsels a wise control of reproduction; but, when I have finished his book, I have almost the impression that such "wise control" would consist in stopping reproduction altogether especially among those who think and lead in the affairs of the world today. The problems of population cannot be solved quite so easily. I am afraid that this problem is too complex for any one person to solve offhand. It will require the sympathetic and thoughtful study of many

minds. When this has been accomplished, I doubt not that the first item in their recommendation will be for birth release among those characterized by the author as "the remaining quarter, who might worthily direct the great majority," and who "sit complacently as long as they are permitted to take an extra toll of the good things of life. . . ." What the other items in their recommendation would be, it would be hazardous to guess.

LOUIS I. DUBLIN

February 29, 1924

Wage Rates in the Bituminous Coal Industry, by W. E. Fisher. Mimeographed; charts and appendices. 1923. 54 pp., 102 plates and tables.

Wage Rates in the Anthracite Coal Industry, by W. E. Fisher. Mimeographed; charts and appendices. 1923. 43 pp., 18 plates and tables.

Earnings of Bituminous Coal Miners, by Anne Bezanson. Mimeographed; charts and appendices. 1923. 50 pp., tables and charts.

The examination of wage rates in the bituminous coal industry by Mr. Fisher is one of several studies made under the general supervision of Dr. J. H. Willits. Mr. Fisher discusses first those salient factors which have contributed to the present wage structure in the industry. He stresses the over development of the bituminous coal industry, the adoption in early agreements of the principle of "competitive equality" (*i. e.*, the fixing of differential rates so that mines in a district where operating conditions are more difficult may continue in competition with mines in a mining area where conditions are more favorable), and the presence of large non-union fields, though the bulk of the industry is unionized. Standardization, he holds, is less prevalent in this than in other unionized industries. As a part of his analysis of the wage structure, Mr. Fisher traces briefly the adoption of bargaining through the Central Competitive Field in 1898 and the emergence of the principle of competitive equality.

About 60 per cent of the workers in the bituminous coal industry are tonnage men, *i. e.*, piece workers. Piece work rates per ton vary considerably. The base rates were fixed originally so as not to disturb the competitive status between the districts, which existed in 1898 when the first four-state agreement was negotiated. Changes since have added a specific number of cents per ton to the base rate. In the union fields in the central competitive area, in all but two districts, 47 cents has been added to the pick mining base rate since 1912. In these two districts the method of payment was changed from a screened to a mine run basis. If the method of payment had not been changed the increase per ton would have been the same as in other districts. Similarly in the union areas outside of the central competitive field the increase per ton since 1912 in nearly every instance has been 47 cents, or approximately that amount. The effect of the agreements reached in the central competitive field on the rates in the outlying districts is thus strikingly shown.

The influence of the union system on wage movements in the non-union areas is almost equally striking, although as Mr. Fisher points out, rates in the non-

union field have changed more frequently and often have been reduced when there were no changes in the union rate. In 37 out of 94 non-union mines in West Virginia, however, the net increase from 1912 to 1923 was between 45 and 50 cents per ton, approximately the same as in the union fields. In sixteen other mines in the same district the change was either from 40 to 45 cents or from 50 to 55 cents. But in this case the range was from 25 cents to as high as 75.6 cents. Moreover, the rise of 75.6 cents in six non-union mines was matched by the same increase in the same period for six union mines in West Virginia and the total tonnage rate in 1923 for both non-union and union mines was the same. The rise in the tonnage rate in the Alabama fields, on the other hand has, since 1912, been much less than in the non-union fields of West Virginia.

The United Mine Workers, Mr. Fisher feels, has adhered much less closely to the principle of the standardized rate than to that of competitive equality. But, he observes (P. 49) that there is "remarkable uniformity both in the trend and the actual rates paid in the union fields for the six inside and the four outside day occupations presented." In another place (P. 17) it is noted that, with regard to union tonnage rates in West Virginia "there has been a distinct tendency to even up rates so that most mines in this district are paying a uniform rate per ton." Again among tonnage workers there has been marked standardization within the same district.

The second of Mr. Fisher's studies deals with the wage rate structure in the anthracite industry. Little in common exists between the bituminous and anthracite coal industries, either in their economic background or in their wage arrangements. The former is highly competitive, over developed, and only partly organized. The latter on the other hand is practically a monopoly, with much unused productive capacity, and almost all the workers are members of the union. Furthermore, geological conditions are such in the hard coal mines as to make necessary a much more intricate and complex system of rates than in the soft coal mines.

The wage structure in the anthracite industry has as its base the rate schedules and trade practices developed in the period of individual bargaining before the strike of 1902. These basic rates and methods of payment were crystallized in the decision of the Anthracite Coal Commission of 1903. With each company, and even each mine superintendent dealing individually with the workers, many different bases of payment and differentials developed. Mr. Fisher presents tables showing the lack of uniformity in the rates. In some instances collieries in 1903 were paying as many as 13 different rates for the same occupation. The record shows the prevalence of similar conditions in 1923.

Mr. Fisher confines his statistical analysis to the rates paid per unit of time employed. He finds such intricate systems of measuring the amount of work for which the piece worker is paid (per ton; per linear yard chamber is advanced; per prop; etc., varying not alone from mine to mine, but in some instances in different parts of the same mine) as to make comparison on any other basis impossible.

In an analysis of the wage rates of 53,159 company men, *i. e.*, workers paid by the hour or day, during the second pay-roll in March, 1923, Mr. Fisher speaks of

the "significant range" in rates as from 27 cents to 71 cents per hour. This range includes over 98 per cent of all the workers reported. But of all these workers 8 per cent of the total and practically all of the group receiving less than 51 cents per hour are boys; and only 2 per cent of all the adult workers receive less than 51 cents. In fact all but five per cent of adult workers receive from 51 cents to 71 cents per hour. In this wage group, however, there is an important division. Over 78 per cent of all adult outside workers receive 51 cents or more but less than 61 cents per hour. On the other hand, over 60 per cent of the adult workers employed inside the mine receive from 61 cents to 71 cents per hour, and 80 per cent of these workers fall in the range of 59 to 71 cents. Mr. Fisher's use of frequency tables with large wage intervals (27 cents to 39 cents; 39 cents to 51 cents; 51 cents to 71 cents; 71 cents and over) is not significant and at times confusing. For instance, in making a comparison of wage rates by coal regions, he is forced (page 6) to append a foot-note to a table, stating that the "wide range of rate intervals in the table covers up the facts."

The discussion of wage changes in the anthracite coal industry from 1902 to 1923 is fully and clearly presented. Actual figures for 1903 and 1923 and relatives for 1903, 1912 and 1923 are given for various occupations and different mines. The figures show that the rate of increase has not been uniform in the different collieries, but in some operations rates have been very nearly standardized. This applies to such important groups as company miners, inside motormen, inside lumbermen, and outside laborers.

The study of earnings in the bituminous coal industry is the most comprehensive of its kind. It covers 1,177 union and 751 non-union mines, located in 23 states. These mines employ, on the average, 332,668 miners. The wage data apply in the main to the year 1921; although some supplementary information for the year 1920 is included, and a comparison is made between 1920 and 1921 for 285 mines. The year 1921 was selected because there was no change in the wage level and no strike in that year. On the other hand the average number days of employment were fewer than usual. In 1920 there was more employment, but in this year there was a wage change about September 1st. Moreover, although 1921 was below the average in volume of employment, the year 1920 exceeded the average, if the war years are excluded. The 1922 figures are not used because of the strike of that year and because the investigation was begun before the end of the year.

Earnings of the tonnage workers, who constitute 60 per cent of the working force, are presented in frequency tables of $100 intervals. There is a large turnover in the working force, hence a large duplication of names. These figures, as Miss Bezanson points out, do not represent annual earnings; for example, a worker may have received $150 from one mine and $1,050 from another. His total earnings for the year would actually have been $1,200, yet he would have been included in this table in the group receiving $100 to $200, and also in the group of $1,000 to $1,100. The table of earnings also gives the average number of "starts" per man, and the average daily wage in each $100 interval. Some idea of the turnover and the consequent duplication of names may be had from

the fact that the largest group of workers appears to have earned less than $100 and that 38 per cent of all workers earned less than $200 in 1921.

Using these data comparison is made between the number of starts required on the average in union fields and in non-union fields to earn certain specified sums. This method of counting is admirably adapted for the purpose of measuring not annual earnings, but the earning power of tonnage rates. They are thus analagous to full time hourly earnings in a piece work industry. The comparative tables show conclusively the higher wage standards prevailing in the union fields. Generally speaking, fewer starts are required in union mines to earn a specified amount than in non-union mines.

It might be urged that actual annual earnings could have been secured by tabulating the earnings of those workers who have been employed by a single mine during the entire year. As a matter of fact earnings are reported for these workers who number 47,000. But for good reasons Miss Bezanson rejects this method of sampling. In the first place this group obtained by and large more employment than those who for one reason or another went from one mine to another or were laid off for a period during the year. Miners not continuously in the employ of one mine during the entire year constitute 75 per cent of the average number of tonnage workers.

Moreover, analysis of the earning data shows that average daily earnings vary with the average starts per pay-roll man during the year. In other words a union machine miner averaging a hundred and fifty starts on the same pay-roll during the year earned about $9.75 per start, and a miner averaging only 75 starts earned about $8.50 per start. This relationship held for all occupations, in union and non-union mines. It is clear therefore that the workers employed in the same mine throughout the year are not a fair sample of the great bulk of mine workers. Figures for this selected group would obviously be too high and not representative.

In the introduction to her report Miss Bezanson states that:

> In a study of earnings in so vast and widely scattered an industry it is not possible to give in a few lines, figures or averages that are either typical or honestly representative. Nothing is gained in either accuracy or truthfulness of presentation by endeavoring to make appear simple that which is essentially complex. Such an effort merely blurs the picture and misleads the reader.

The data referring to all tonnage workers are therefore presented in frequency tables. There is no doubt much force in this comment, but analysis by the means of accepted statistical devices such as averages may do much to bring out significant and relevant information.

Pay-roll data for tonnage workers are given in two summary tables, one for the 1,177 union mines, the other for 751 non-union mines. These tables show the number of workers, classified in four occupational groups and the total in all groups, reported as having received from any one mine during the year earnings, classified into $100 intervals. The total number of starts, the average starts per man, and the average daily earnings for the workers reported in the earning interval are also given.

Altogether 14,653,909 starts (days employed) are reported for all workers in

the union mines. Of this number 8,493,812 starts are reported for loaders. In over one half of the starts made by loaders the average daily earnings were between $6.50 and $7.52. The modal earnings group using starts as the unit was $700 to $1,000 for the year. The three $100 intervals: $700 to $800, $800 to $900, $900 to $1,000, account for the same number of starts. The median group is $800 to $900. In this earning interval the average daily earnings are $6.97; the average number of starts 122. The upper quartile range is $1,200 to $1,300 with average daily earnings of $7.96 and average starts of 161. Further analysis of the data for other occupations and total workers in the union mines and for the non-union mines shows similar results. The scatter is not as great as in many other industries.

Miss Bezanson makes a fuller analysis of the reports of earnings of men employed throughout the year at the same mine. This analysis is made by district as well as for union and non-union mines. The schedules and letters of instruction are included in an appendix.

These reports of the United States Coal Commission have not been printed. They have been issued in mimeographed form. There has been no more scientific and extensive investigation of wage rates and earnings in any industry in this country. The reports should be printed and made available for students and others who wish to learn the facts about an important basic industry.

H. K. Herwitz

New York City

Studies in the Economics of Overhead Costs, by J. Maurice Clark. Chicago: University of Chicago Press. 1923. xiv, 502 pp.

Professor Clark's treatment of the subject of overhead costs—"costs that cannot be traced . . . to particular units of output" or costs which "do not vary with output"—is remarkably comprehensive and exhaustive. He emphasizes the fact (in Chapter II and elsewhere) that such costs are by no means confined to particular industries or to particular classes of productive factors, but are present in some form throughout the industrial structure. He shows that overhead is not restricted in its significance to the individual producer or to the specific business concern, but is a matter of the utmost social importance, particularly in connection with the problems of unemployment (Chapter XVIII) and of general business instability (Chapter XIX). He presents a careful statement (in Chapters IV to VII) of the principles governing the utilization of productive capacity for business in general and supports this statement (in Chapters XIII to XVII) by special studies of the transportation and public utility fields and some examination of the problem of cost and capacity in manufacturing, merchandising, and other lines. In addition attention is directed to the underlying functions and methods of cost accounting, there is an interesting discussion of the kinds of business rhythms, and special chapters are devoted to discrimination, cut-throat competition, and governmental costs. In a concluding chapter, further, the author undertakes to make some application of his findings to the laws of value and distribution.

In Chapter III and elsewhere the author does good service in stressing the many-sided character of the cost problem. It is pointed out that there are many classes or kinds of cost, each of which has a more or less independent validity, and that no one system of accounting or statistical method can be expected to answer all of the questions with respect to cost in which business managers or others may be interested. This is emphasized by the illustrative case discussed in Chapter IX, in which the author states "nine typical problems," each of which requires a somewhat different solution and in several of which the underlying data required vary. An examination of this case should have a salutary effect upon the opinions of the accountant who is inclined to feel that all cost questions of any moment can be adequately answered by an analysis and compilation of ledger entries. In this illustration (and throughout the book) the significance of differential cost—the cost of a specific increase or decrease in output—is insisted upon.

In the discussion of the underlying purposes and methods of cost accounting the author exhibits a keen insight and a sane attitude with respect to moot points. He recognizes, for example, that it is unnecessary that hypothetical interest charges be actually included in the books of account, although he holds, quite properly, that in reaching sound decisions in connection with certain kinds of questions business managements must take cognizance of normal return on investment. Again, he recognizes that prices cannot be absolutely determined by cost accounting, without reference to market conditions, but he points out that cost data can be of marked assistance to the management in the formulation of rational price policies. On the other hand Professor Clark may perhaps be accused of exhibiting a little too much optimism with respect to the possibility of developing methods of cost finding which will furnish dependable answers to the kinds of questions which he raises. In the judgment of the reviewer several of the "nine problems" are in the nature of the case incapable of any precise, oreven approximate, solution; answers to such questions will always be conjectural.

Professor Clark takes the stand (page 197 and elsewhere) that true cost is measured by the market price of raw materials rather than by their original cost. This position is to be commended, but the essential reason therefor—the fact that replacement costs in most cases have more influence upon competitive selling prices than original costs—appears to be neglected. Further, the doctrine that inventories should be priced at actual cost, which the author seems to support (page 240), is not consistent with this stand. From an accounting point of view, likewise, there is serious objection to the exclusion of depreciation from operating expenses (page 192), and to the assumption that the periodic net income which the financial accountant seeks to discover is the amount available for dividends (page 192 and elsewhere).

The discussion of the different kinds of business rhythms is especially interesting. It is pointed out that the daily, weekly, and seasonal ebb and flow in the demand for various classes of goods and services is founded in considerable measure upon human habits and requirements, and that no complete elimination of these irregularities is possible or even desirable. Some consideration is given,

nevertheless, to methods and policies designed to modify the severity of these fluctuations in demand.

It is in dealing with the problem of unemployment (especially as resulting from the business cycle) and general business instability that Professor Clark emphasizes the importance of methods and agencies of control. He shows that from the standpoint of industry as a whole the ultimate cost of labor is made up largely of elements which are in the nature of overhead, and emphasizes the undesirability of allowing the burden to fall entirely upon the laborer.

> The necessity of letting . . . idleness take the form of "unemployment" as we know it—meaning that the worker bears the burden of his own idle time . . . amounts to assuming that industry cannot be made to bear the full responsibility for all the overhead costs of which it receives the benefit. The result is that those who can do the most to regularize industry do not have the incentive, because they do not bear the worst of the costs of which irregularity is the cause. (Page 369.)

Among the remedies from the employers' standpoint are noted the placing of the laborer upon a salaried basis, the guarantee of a minimum of employment per period, manufacturing for stock, the "dovetailing" of seasonal lines, and the payment of a bonus on dismissal or during a lay off.

The author's tendency to assume that competition is inevitably suicidal in the absence of "agreements, understandings, and the sentiment against 'spoiling the market,'" may perhaps be questioned. Is it reasonable to assume that producers are inclined to consider only differential costs in their contest for business, and that unless checked by the trade mores they will accordingly plunge into bankruptcy? Is it not more nearly true that the controlling idea in the producer's mind is the importance of covering total costs and leaving a margin for earnings? In other words, is not the producer in general much more fully influenced by averages than by differentials? Likewise, it may not be unreasonable to suggest that, much as it is desirable to discover and analyze the considerations which control the conduct of business concerns during the various stages of the business cycle, it does not appear from the findings to date that serious modification of the generally accepted doctrines of value and distribution are thereby required.

This book is indeed a noteworthy piece of work. It is marked throughout by breadth of view, wide information, acute analysis, and sound deductions and recommendations. And it is very well written. It will be found of real value by all who are interested in either the general or the technical aspects of cost analysis.

W. A. Paton

University of Michigan

Railroads; Rates—Service—Management, by Homer Bews Vanderblue, Ph.D., and Kenneth Farwell Burgess, LL.B. New York: The Macmillan Company. 1923. xv, 488 pp.

The railroad problem expands most appallingly, and the difficulty of giving it adequate treatment within the covers of a single volume increases in corresponding measure. The present book limits itself to a study of the problem of public

control, and differs from the usual study in paying less exclusive attention to rates, and undertaking to give substantially equal attention to matters affecting rates, service and management. The problems of rates, however, still occupy more space than either of the other two major divisions, even without allowing for the fact that valuation is treated under the heading of management—though its relation to rates is more obvious and more important in most people's minds. Most readers will probably agree that the authors have achieved a very just apportionment of space and emphasis. This feat argues a very comprehensive knowledge of the field, enabling the authors to rise superior to the common temptation to concentrate on those topics with which they are most familiar or in which materials are most accessible, rather than on those which are most important.

The book is, in fact, marked throughout by an extremely broad and competent knowledge of the subject, rounding it out by an adequate treatment of some topics which other books neglect, and making a notable and useful addition to the literature of railroad control. It is an informed and informing book. The text is a triumph of condensation, while the voluminous footnotes expand some points which the text treats too briefly for the serious student, and furnish a mine of references for further study: chiefly in cases before the Interstate Commerce Commission and the Federal courts. The combined backgrounds of the two authors—one a professor of transportation and the other general attorney for the Burlington—bring together a combination of keen analysis, systematic organization and first-hand experience; and the result impresses the reviewer as superior, in a number of respects, to anything of its kind he has yet seen.

Among the special features of the book, Mr. Burgess is presumably responsible for the description of commission organization and procedure, while the very timely analysis of the actual valuations thus far put forth by the Interstate Commerce Commission it is natural to attribute to Professor Vanderblue. There is also a description of the regional rate structures in the United States which is at once comprehensive, condensed and concrete—the most enlightening brief survey to be found in the general literature of the subject. Eight chapters are devoted to service, covering the obligation to serve, regulation of safety and health, trains and train movement, car supply and distribution, through routes and routing of freight, terminals, special privileges and facilities, and new construction and abandonments. Seven chapters are devoted to management, taking up the function of management, the rehabilitation of credit, valuation, the protection of investors, the adjustment of labor disputes, accounts and consolidation. There is no attempt in the text to trace the historical development of regulation, but one of the appendixes gives a very brief chronological table of Federal legislation (anti-trust laws not included).

In discussing valuation, the authors puncture the fallacies of commercial value, and set up in its place the concept of "invested value," but without elucidating this phrase at sufficient length to show just what it would include, and how it would avoid the difficulties which attend the measurement of "original cost." The chapter concludes with this sentence: "A rapt attention to a demonstration of the inconsistencies of the Valuation Act is liable to leave the critic in the posi-

tion of those, who, in the sixteenth-century, demonstrated the flatness of the earth while adventurous spirits were actually sailing toward the West to reach the East."

The high quality of the book justifies the critic in noting defects of detail which might be passed over in a less significant work. On page 63 is the statement that the act of 1906 gave the Commission the power to fix rates only after investigation made upon the Commission's own initiative, whereas the power was limited to cases brought on complaint. On pages 88–89, after stating that added traffic may call for new cars or locomotives, or even new trackage, one reads: "But competent authorities have estimated that about 55 per cent of railroad expenses go on uniformly, regardless of the volume of business done." This gives the impression that, even when trackage is being enlarged to accommodate increased traffic, 55 per cent of the expenses are unaffected by such enlargements; but the authors state immediately afterward that this formula holds only while there is unused capacity. The whole question of regional classifications and the movement toward uniform classification is passed over in the text, which does not even call to the reader's attention the fact that two footnotes give the subject some slight treatment.

On page 108, referring apparently to the rate increase case of 1911, the statement is made that the Commission accepted, among other reasons for refusing to grant increases, theoretical possibilities of economies in operation, apparently referring primarily to those urged by Louis D. Brandeis. Yet in its opinion on this case the Commission specifically stated that such innovations were beyond its province: it might require reasonable economy, but could not direct improvements nor enforce economies. It was not the management of the roads. In its estimates of future earnings, the Commission counted on a continued improvement of efficiency, because experience justified the expectation, "but perhaps not to the same extent as in the past." (XX, I. C. C. R. 285.) It did not take Brandeis' claims at face value, nor give them any clearly definable weight.

One question prospective readers will ask is: Is this an impartial scientific work or a brief for the railroads, from the law department of the Burlington system? And if intended as a scientific work, does the cloven hoof of the railroad attorney show beneath the academic gown? The answer appears to be that the work is both scientific and honest, being largely descriptive rather than critical; concerned with what has happened and what is likely to happen rather than what ought to happen. The authors take no striking stands on hotly-controverted issues, and maintain a balanced tone. They waste no breath protesting against any *fait accompli*, already sanctioned by the Federal power, though they do pay their respects to the more demagogic of the state laws. They do not go out of their way to furnish ammunition to the critics of the railroads. They accept nothing that is not a *fait accompli*, and they do express the railroad point of view on those matters which lie on the disputed frontier of control.

For instance, they cast serious doubt upon the constitutionality of the partial recapture of earnings provided for by the Transportation Act of 1920, stating that it will require judicial genius to square this taking of earnings with the Fifth Amendment to the Constitution. The court has since demonstrated its judicial

genius, upholding the recapture provision in a unanimous decision. The authors characterize this provision as an application of Henry George's economics and one which discriminates against railroads. Many economists might be prepared to admit the first charge without serious perturbation; but the second will take a deal of proving, since the basis of the discrimination already exists in the public nature of the railroad business.

The authors note the fact that the raising of railroad capital has of late years depended almost exclusively on bond issues, but do not trace the responsibility for this unfortunate condition. In fairness, the blame must be shared between the regulating authorities and the railroad managements themselves. In discussing the evils of postponed maintenance the position is taken that the sole public interest in this case arises from the possible resulting breakdown of plant and service. Yet surely this is too narrow a view, for the public is interested in fair earnings on the *maintained and unimpaired* investment in the properties. In discussing the settlement of labor disputes little attempt is made to estimate the real worth and weight of the labor point of view.

In any study of the public control of transportation, in or out of the classroom, the student needs material from more than one type of source. He should get an insight into the problems which face the management, such as only those actually engaged in the business can give in the most realistic way; and he should counteract the natural bias inseparable from such a point of view by other material from sources which have no interests at stake, and which have a correspondingly less intimate knowledge of the facts. In such a combination of materials the present volume should take high rank. It presents a maximum of information with a very moderate degree of bias, and while it should not be taken as, in itself alone, a sufficient introduction to the transportation problem, it requires remarkably little supplementing.

J. M. Clark

University of Chicago

Costs and Profits; Their Relation to Business Cycles, by Hudson Bridge Hastings. Boston: Houghton Mifflin Company, 1923. (Publications of the Pollak Foundation for Economic Research, No. 3.) x, 168 pp.

This study is an attempt, by means of a circumstantial step-by-step description of the monetary and financial aspects of the mechanism of the productive process, to demonstrate that during the upswing of the business cycle the value of goods produced at the current price level exceeds the flow of purchasing power into the hands of ultimate buyers, that the rate of production therefore exceeds the rate of consumption, with a consequent increase in inventories of producers and dealers, and that it is this overproduction so brought about which causes the ensuing business depression. If producers were not to allow profits to accumulate as they do during a period of business expansion, but were promptly to distribute them to shareholders, or invest them in expansion of plant, there would be an approximate equalization of the inward and outward flow of money, stocks would not accumulate, and there would presumably be little or no cyclical fluctuation in business.

The validity of this thesis is dependent upon the accuracy and completeness of the author's description of the sequence of events during the upward trend of business. The accuracy of his account is beyond doubt capable of being subjected at almost every step thereof to inductive verification, if not from complete statistical data, at least from the accounting and other records of many representative producing and distributing concerns and of the important banking institutions. The necessary data for verification purposes probably exist in large measure in systematic and readily available form. But the only concrete data presented in the entire book are used solely for illustrative purposes and consist of the balance sheets of three moderate-size concerns "chosen at random" and with "no attempt . . . to determine to what extent the mathematical results in these three cases are typical of all business and industry." The author's description of the mechanism of the industrial process must be considered, therefore, as of unproven accuracy, and as a rather uncertain foundation for as elaborate a superstructure as he ventures to construct upon it.

But even if the accuracy of Professor Hastings' description, as far as it goes, be accepted without reservation, is it complete enough in the factors it takes into consideration to justify the conclusions he derives from it? The author declares that during the upswing of the business cycle there is an excess in the inflow of money to the producer of consumers' goods over his money expenditures, and concludes that the flow of money into the hands of final consumers must therefore be at a rate inadequate to take up the flow of goods from producers at the current price level. Ignoring the fact that it is difficult to conceive in modern times of money accumulating in the cash boxes of producers, and that failing such accumulation all money coming into possession of producers must almost immediately pass on (through devious paths perhaps) to ultimate buyers, let me present two other hypotheses, rivalling the author's in plausibility, but leading to different causal explanations of the business cycle. The author either wholly overlooks these hypotheses, or he mentions them only to brush them cavalierly aside.

(a) During a period of expanding business the increase in railroad, highway and building construction and in the production of manufacturing equipment, rolling stock, shipping and other capital goods, leads to a rate of outflow of money from the concerns engaged in these enterprises to final consumers greatly in excess of the inflow of money to such concerns from final consumers. The more rapid increase of purchasing power in the hands of final consumers than of consumers' goods available for their purchase results in a rise in the prices of such goods until consumers or retail dealers, irritated by this rise in prices and anticipating a reversal of the price trend, "strike" and thus bring an end to business expansion.

(b) In our complex industrial, financial, and monetary organization, there tend to be during the upswing of the business cycle lags at different points in the process between the inward and outward flow of money. As long as there is general confidence in the continuance of the upward trend of prices and of business prosperity, these lags are prevented through the injection of bank money at the points where there tends to be a deficit, this bank money either being drained off from the points of surplus funds or, where this does not suffice, being

newly-created through the extension of bank credit. Only when business confidence departs, so that new credit ceases to be granted, or is granted only at a diminished rate and sparingly, do such lags actually appear at any point in the process and only then do they operate as a check upon the continuance of prosperity.

The author has presented an ingenious theory, and he has succeeded in giving a new twist to a general overproduction theory of the business cycle, which keeps it free from obvious logical fallacies and makes it dependent for its validity upon facts rather than syllogisms. He fails, however, to present the supporting facts. He will have completed his task only if he proceeds to discover whether the facts are with him or against him.

JACOB VINER

University of Chicago

Introduction to Medical Biometry and Statistics, by R. Pearl. Philadelphia: W. B. Saunders. 1923. 379 pp.

A book on this title by Pearl might well draw forth anticipations of something especially valuable and original; and this anticipation is not disappointed. As the author points out, his book is written primarily for the medical man and the illustrations of methods are mainly chosen from that field. The tabulation of statistical data, and their graphical representation are treated in turn. Rates and ratios are discussed and the topics elaborated in their relation to mortality statistics. The probable error concept is treated and the theory of probability explained. The measurement of variation and correlation are then considered and there is a final chapter on simple curve fitting. Five appendices of mathematical tables and formulae complete the book.

It is hard to find anything for this book except praise. Mathematical conceptions which would repel if developed by the language of mathematicians are here made clear by the use of delightful English. Such a text book which grows out of a professor's set of lectures is pretty sure to have been purified of obscurities and methods of treatment which are beyond the training of the student. The only thing the reviewer finds to criticize adversely is a tendency to overdevelop a subject, a certain verbosity at times, as in the chapter on the elementary theory of probability where the toss of a penny is discussed. It is all delightful as literature, however, and the section would make a good magazine article. With his well-known courage Pearl applies biometry to various medical data and is often led to conclusions opposed to current prejudices. He has a confidence in the statistical method and a willingness to accept the conclusions to which the method leads.

There is no doubt of the usefulness of the book. Medicine seems woefully short of good statisticians; there is too much of basing conclusions on a limited experience in which the last few cases bulk very large. Every man who is in training as vital statistician, health officer, epidemiologist ought to study this book and few medical men can afford to be ignorant of the conceptions and methods it makes clear.

CHARLES B. DAVENPORT

Cold Spring Harbor

The Pre-War Business Cycle, 1907 to 1914, by William Charles Schluter, Ph.D. Columbia University Studies in History, Economics and Public Law, Vol. CVIII, No. 1 (whole No. 243). New York. 1923. 189 pp.

The main purpose of this monograph is to determine the causes of the business depressions of 1910–11 and 1913 in the United States, and of the "premature occurrence of those depressions as related to the normal ebb and flow of general business conditions abroad." Dr. Schluter proceeded by the statistical method, and has without question produced a comprehensive and painstaking study.

The conclusion, however, is in no sense novel. Dr. Schluter definitely aligns himself with those writers who consider bank credit practically the sole determinant of changes in the tide of business; other forces, in so far as they are effective at all, take effect only through bank credit.

"The foregoing analysis, based upon our study of the period 1908 to 1913, seems to indicate the following conclusions:

a. That the limits of elasticity in the credit system, defined on the one hand by the minimum of credit requirements in depressions, and, on the other, by the amount of bank reserves regulate the spurts in prices and production, namely, the lengths of the business cycles herein examined.

b. That the relation between the normal trend of agricultural production and industrial development during this period was one of the several important factors which affected the relatively permanent requirements of credit, and the supply of bank reserves.

c. That the moment when the two cycles of 1908 to 1911 and 1911 to 1913 attained their zenith and their nadir was determined chiefly by the supply of bank credit and the use made of it."

Logically, the book may be said to comprise three parts. The first, composed of Chapters I, II, III, V, and VI, contains a detailed recital of the events preceding and accompanying the depressions of 1910–11 and 1913. The second, including Chapters IV and VII, explains the causes of the two depressions considered separately. The third part, Chapter 8, expands the reasoning into a general theory as to variations in the length of business cycles.

The argument would have been far more convincing, in the judgment of the reviewer, if the author had adopted a somewhat different method of exposition. He devotes one chapter to "The Depression of 1908," another to "The Revival of 1909," a third to "The Recurring Depression of 1910–11," etc., and fills each with numerical and factual evidence of a very detailed sort. Therefore the critical reader has difficulty in finding the statistical background for interpreting these isolated figures; he is forced to search the rest of the book for statistics given in a sufficiently long series to make possible a study of the fluctuation in question. This inadequacy of organization is illustrated by the author's references to production. The only continuous statistics on industrial production are given in Chapter II, "The Revival of 1909," and there is covered the entire period for 1905 to 1914, despite the chapter title. All of the statistical evidence could have been more efficiently marshalled in one or two introductory chapters. The entire period 1907–14 (and preferably also a few years before and after) could have been so presented before the detailed analysis of the critical phases in the cycle. No small advantage of such a method is that it would have made logically possible a freer use of charts. In its present form the book contains only

one—a defect which is not entirely counterbalanced by the writer's exceptionally skilful use of tables. From the standpoint of exposition the book is easily excelled by some other studies of cyclical fluctuation over short periods, notably Hansen's "Cycles of Prosperity and Depression . . . 1902–1908."

Aside from presentation the only defect in the work is the writer's tendency (not only in the final chapter but also elsewhere) to generalize too broadly from the limited experience of the two cycles covered by his study. At times he is evidently trying to guard against this by suitably qualifying his statements (*e. g.* in the excerpt quoted above); but on many occasions he relapses into a dogmatic statement in which he seems to have omitted qualifications not to avoid repetition but because he intended to state a universal proposition. The author seems clearly to have derived—either from previous consideration of the subject or from further empirical study of other periods and other countries—a much more general theory as to the unique position of bank credit in the business cycle than the mere evidence here published would justify. This raises the question whether there are not other potential limiting factors than bank credit, one or another of which might, under different circumstances, become operative before the strain upon bank credit could do so. The course of trade and industry during 1923–24 certainly raises this issue in poignant form. Here there has been no question of the adequacy of bank credit. Is it not possible that several other factors (cost of production as one good example) may come to set the actual limits upon business expansion, and in quite other ways than through bank credit?

Dr. Schluter's study is as a whole stimulating and in many ways convincing; it is decidedly worth the reading of any serious student of business economics.

W. A. Berridge

Brown University

Climatic Changes. Their Nature and Causes, by Ellsworth Huntington and Stephen T. Visher. New Haven: Yale University Press. 1922. 329 pp.

Professor Huntington began with his "Pulse of Asia" (1907) to attract a reading public of his own. His many readers and friends will not be disappointed at this latest effort. He and Professor Visher have scored again in popularizing the geographic sciences. The success of Huntington, Cushing, Williams, and Visher in creating for themselves an audience which reaches from the secondary school student, through the colleges, then to the family library table should excite a desire to emulate them on the part of our statistical brethren who have just as much to write about and who have, in fact, the same sort of audience. Huntington's work and its consequences reminds us of the wave of interest in the natural sciences generated by Huxley and by Richard Anthony Proctor during the '80's in England. When will statisticians realize that they too have an opportunity to bring their work home to the American people? Must they be told that statistics and its results should be prepared from the standpoint of the consumer and not that of the compiler? Certainly, statistics and the statisticians can become as interesting as geography and the geographers! Or, take mathematical physics: when Niels Bohr and Albert Einstein can attract audiences that tax the capacity

of university halls, the statisticians surely ought to be able to reach more of the people than are represented in the membership of the learned societies. Has statistics passed its "Golden Age"? Were Anthony Ashley-Cooper, Florence Nightingale, Francis Amasa Walker, and Carroll D. Wright the last of the statisticians whom the public understood?

In the opening chapter, Huntington and Visher discuss the uniformity of the earth's climate. They recognize "a uniformity so pronounced and of such vast duration as to stagger the imagination." Superposed upon this uniformity there seems to be a tendency toward complexity, and this tendency has been broken by pulsatory changes, first in one direction, then in another. The average temperature of the earth's surface, they say, has never varied more than perhaps 30 degrees C. above or below the present level.

Then, in Chapter II, the authors discuss the variability of climate. The several types of climatic sequences are distinguished as follows:

1. Cosmic uniformity
2. Secular progression
3. Geologic oscillations
4. Glacial fluctuations
5. Orbital precessions
6. Historic pulsations
7. Brückner periods
8. Sunspot cycles
9. Seasonal alternations
10. Pleionian migrations
11. Cyclonic vacillations
12. Daily vibrations

Having defined each of these terms, and incidentally, having set up trains of thought for statisticians, our authors proceed to examine certain hypotheses of climatic changes. Then there is a chapter on the climate of history, wherein the pulsatory hypothesis is tested by an appeal to the facts on the growth of trees. Changes in California climate for 2,000 years, as measured by the growth of Sequoia trees, are set forth graphically on page 77. Then an attempt is made to determine how accurately the growth of trees represents the local rainfall. The authors say that "perhaps the best way to determine this is the standard mathematical method of correlation coefficients." This is followed by the curious statement that when "a correlation coefficient is four times the probable error, . . . it is generally considered to afford evidence of some kind of relation between two phenomena. When the ratio between coefficient and error rises to six, the relationship is regarded as strong." On page 80, the authors give a table of observations, coefficients, and their errors, based upon rainfall and tree-growth records of California trees. In each "correlation" the ratio r is given. Perhaps the authors really meant to say that r was a convenient measure of *dependability* so far as the *number* of paired observations was concerned. Some test of linearity seems to be in order before one can accept these correlations which are used to show pulsatory changes in climate.

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That there is work for the statistician is suggested by the authors' emphasis in the closing chapter on the need for a greater amount of quantitative testing of hypotheses of climatic change. They hold that the next great step is to ascertain whether the forces identified as factors in climatic change are actually great

enough to produce the observed results. A fuller comparison between earthquakes on the one hand and meteorological conditions and the wandering of the poles on the other, is called for in their opinion.

> "Throughout the vast eons of geological time the adjustment of force to force, of one body of matter to another, and of the physical environment to the organic response has been so delicate, and has tended so steadily toward the one main line of mental progress that there seems to be a purpose in it all. If the cosmic uniformity of climate continues to prevail, and if the uniformity is varied by changes as stimulating as those of the past, the imagination can scarcely picture the wonders of the future. In the course of millions or even billions of years the development of mind, and perhaps of soul, may excel that of today as far as the highest known type of mentality excels the primitive plasma from which all life appears to have risen."

Professor Huntington has in recent years been responsible for a revival of interest in the subject of climate in its relation to the health, longevity, and efficiency of mankind. His studies, especially those on influenza, suggest that the geographers, epidemiologists, and statisticians, need to work together more closely in the future than in the past. One group cannot carry the burden of these necessary studies of the effect of seasonal and daily variations in weather elements on the health of man. From the initiative of Professor Huntington there has arisen more or less productive effort by Committees of the American Meteorological Society, the American Public Health Association, and the Ecological Society of America. Teachers of statistics in our schools of public health should endeavor to arouse more interest than has hitherto been shown in applied climatology. Research funds can be made available, and all that is needed for the development of a neglected subject in this field of statistics is the lively interest of teachers who direct public health and meteorological research.

E. W. Kopf

New York City

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CONTENTS

THE MEASUREMENT OF CORRELATION AND THE PROBLEM OF ESTIMATION. By FREDERICK C. MILLS 273

AN IMPROVED METHOD FOR MEASURING THE SEASONAL FACTOR. By WILLFORD I. KING 301

IS THE FAMILY OF FIVE TYPICAL? By PAUL H. DOUGLAS 314

A NEW CLEARINGS INDEX OF BUSINESS FOR FIFTY YEARS. By CARL SNYDER 329

REVISION OF THE WEEKLY INDEX NUMBER. By IRVING FISHER. 336

THE RELATION OF THE UNITED STATES TO INTERNATIONAL STATISTICS. By WALTER F. WILCOX 348

NOTES:

THE HIGHER COST OF FOOD IN MASSACHUSETTS. By Robert L. McFall 362

WHOLESALE PRICES, 1801–1840. By Alvin H. Hansen 377

MONTHLY PIG IRON PRODUCTION, 1884–1902. An Analysis of a Problem in Correlation. By Holbrook Working 381

DIVORCE RATES, 1922. By Walter F. Willcox 387

SOCIAL SCIENCE ABSTRACTS (390); TRADE STATISTICS (392); PROGRESS OF WORK IN THE CENSUS BUREAU (396); PROFESSIONAL AND EDUCATIONAL STANDARDS (399); MISCELLANEOUS NOTES (404).

REVIEWS: Wilkinson: *Wages in the Coal Industry*, by Anne Bezanson (410); Ely: *Railway Rates and Costs of Service*, by M. O. Lorenz (412); Whittaker and Robinson: *The Calculus of Observations*, by Arne Fisher (413); *Tenth Annual Report of the Federal Reserve Board*, by H. Parker Willis (417); Lubin: *Miner's Wages and the Cost of Coal*, by R. F. Foerster (419); Falk: *Principles of Vital Statistics*, by Edgar Sydenstricker (420); *Commerce Year Book 1922*, by F. A. Ross (421); *Mortgages on Homes*, by J. M. Gries (423); *The American Labor Year Book*, by P. H. Douglas (424).

RECENT LITERATURE 426

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